HETEROSCEDASTICITY AND EFFICIENT ESTIMATES OF BETA

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This study investigates the presence of conditional heteroscedasticity in the market model residual terms and the efficiency of beta estimates. Nonnormality and heteroscedasticity in the market model residual terms make the estimators inefficient and some of the significance tests invalid. An extension of the Autoregressive Conditionally Heteroscedastic (ARCH) model, the Bollerslev's Generalized Autoregressive Conditionally Heteroscedastic (GARCH) model, is applied to a sample composed of securities traded at the Budapest Stock Exchange, which allows us to test whether the conditional heteroscedasticity, mainly observed in the United States market, is also present in the Hungarian stock market.

KEYWORDS: Conditional heteroscedasticity; Beta estimates; GARCH models.

In the terminology of the capital asset pricing model (CAPM), beta is a measure or price of risk that arises from the reasonable and widespread idea that changes in stock returns are directly related to market changes. It is the difference between the expected rate of return on market portfolio and the riskfree rate of return. The equation describing this relationship has been developed by *Sharpe* (1963) and is known as the *market model*. The market model is a simple statistical model which relates the return of any given security to the return of the market portfolio. The model's linear specification follows from the assumed joint normality of asset returns. For any security i we have

$$R_{it} = \alpha_i + \beta_i R_{mt} + \varepsilon_{it}$$
$$\mathrm{E}\left[\varepsilon_{it}\right] = 0 \qquad \mathrm{Var}\left[\varepsilon_{it}\right] = \sigma^2_{\varepsilon_i}, \qquad /1/$$

where

R_{it} is the random return on stock i in period *t,*

R_{mt} is the random return on the market index in period *t,*

α_i is the component of stock i's return that is independent of the market performance,

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β_i or beta is the measure of the expected change in R_{it} given change in R_{mt},

ε_{it} is the random disturbance term with an expected value of zero and variance of $\sigma^2_{\varepsilon_i}$.

Equation /1/ is frequently used to forecast stock returns. As the future returns are unknown, in practice it is necessary to rely on estimates of the model parameters based on historical data, that is

$$\hat{R}_{it} = \hat{\alpha}_i + \hat{\beta}_i R_{mt}, \qquad /2/$$

where R_{mt} denotes the actual return of the market index regarding it as the proxy of the market portfolio, $\hat{\alpha}_i$ and $\hat{\beta}_i$ are the estimates of α_i and β_i respectively.

When using the ordinary least squares (OLS) technique, which generates best linear unbiased estimates (BLUE), the beta estimates are given by the following formula

$$\hat{\beta}_i = \frac{\sigma_{im}}{\sigma^2_m} = \frac{\mathrm{Cov}(R_i, R_m)}{\mathrm{Var}(R_m)}, \qquad /3/$$

widely used in finance. In the Sharpe model the endogeneous variables (individual returns) are not independent, what is more they partly compose the exogeneous variable (market portfolio return). Thus, this is a multivariate regression model consisting of non-independent equations. Estimating these equations separately, the estimates probably will contain certain SUR bias, but it should also be remembered that in the original Sharpe model the market return is present and not the value of the market portfolio (or as its proxy the index value) and the market return is not a linear combination of the individual returns.

The ordinary method assumes that the disturbance term is *white noise*, that is, conditions of normality with zero mean, finite and constant through time (homoscedastic) variance, and universal uncorrelation are hold. However a number of studies have raised questions on the validity of the market model to estimate the systematic risks of financial assets using the OLS technique. It has been shown that some of the assumptions such as homoscedasticity do not always hold. The most important implications of heteroscedasticity are:

1. The OLS estimators will be inefficient, since they will not have the minimum variance in the class of unbiased estimators. This fact can partly explain the nonstability of beta estimates and makes impossible to use past values of betas for forecasting their future values. So the accuracy of beta estimates also can not be evaluated in a correct way. (*Blume*; 1971, *Levy*; 1971, *Theil*; 1971, *Lin–Chen–Boot*; 1992).

2. Significance hypothesis tests of the estimates will be performed with a higher type I error than it is assumed, since the estimated covariance matrix will be biased. In similar way, other tests, based on homoscedasticity, e. g., the Chow test for parameter stability will no longer be valid.

3. The coefficient of determination R^2 will decrease, wich means that systematic risk will be understated, while diversifiable risk will be overstated. As *Fisher* and *Kamin* state (*Fisher–Kamin*; 1985, p. 129), errors in beta estimates are the equivalent of extra non-systematic individual risks.

For these reasons, it is necessary to take heteroscedasticity explicitly into account. Although many of the previous studies consider it in the CAPM tests, only a few exceptions investigate this question in the estimation of betas with the market model. *Miller* and *Scholes* (1972), *Brenner* and *Schmidt* (1975), *Martin* and *Klemkovsky* (1975), *Belkaui* (1977), *Brown* (1977), and *Bey* and *Pinches* (1980) find evidences of heteroscedasticity in the market model. The previously listed authors use a wide variety of methods: from simple analysis of scatter diagrams and regressions, to the Bartlett, the Glejser, or the Goldfeld-Quandt tests. However, *Giaccotto* and *Ali* (1982) point out that unconditional acceptance of that evidence can not be advisable, among other reasons, because the tests are not reliable if regression residuals are non-normal. This is a very common case, as probability distributrion of asset returns are usually markedly leptokurtic (see, for example, *Varga*; 1998). But apart from this evidence, rarely has literature dealt with the estimation of beta explicitly considering heteroscedasticity. We mention the following exceptions. *Schwert* and *Seguin* (1990) apply the weighted least squares (WLS) technique, instead of the OLS one, to estimate betas. This procedure requires the introduction of an exogenous variable – normally, the market return – in order to predict the residual variance and takes into account unconditional heteroscedasticity. *Bera*, *Bubnys* and *Park* (1988), *Diebold*, *Im* and *Lee* (1988) and *Morgan* and *Morgan* (1987) use the Autoregressive Conditionally Heteroscedastic (ARCH) model of *Engle* (1982), that is, they estimate betas considering residual variance of today depending upon yesterday's error. This model is used by Schwert and Seguin, who find similar results to those of the WLS regression. Finally, *Corhay* and *Rad* (1996) apply a market model which accounts for GARCH (Generalized Autoregressive Conditionally Heteroscedastic) effects.

The following part of this study is divided into four sections. First, the applicability of the GARCH models to capture the serial correlation of volatility in financial time series is discussed. The second section presents the data used for model specification. In the third section the empirical findings of normality and heteroscedasticity are presented and discussed. The final section of the paper contains brief conclusions.

THE GARCH MODEL

In order to concentrate on volatility of a time-series ξ_{t+1}, we assume that ξ_{t+1} is an innovation, that is, it has zero mean conditional on time *t* information. In an application in finance, ξ_{t+1} might be the innovation in an asset return. We define σ_t^2 to be the time t conditional variance of ξ_{t+1} or equivalently the conditional expectation of ξ_{t+1}^2. It is also assumed that conditional on time t information, the innovation is normally distributed: $\xi_{t+1} \sim N(0,\sigma_t^2)$. The unconditional variance of the innovation, σ^2, is just the unconditional expectation of σ_t^2. (For a series with a time-varying conditional mean, the unconditional variance is not the same as unconditional expectation of the conditional variance. This result holds only because we are working with an innovation series that has a constant (zero) conditional mean).

To capture the serial correlation of volatility in financial time series, *Engle* (1982) proposed the class of ARCH models. These regard conditional variance as a distributed lag of past squared innovations:

$$\sigma_t^2 = \omega + \theta(L)\, \xi_t^2, \qquad /4/$$

where θ is a polynomial in the lag operator. To keep the conditional variance positive, ω and the coefficients in $\theta(L)$ must be non-negative.

As a possible way to model persistent movements in volatility without estimating a large number of coefficients in a high-order polynomial $\theta(L)$, *Bollerslev* (1986) suggested the GARCH model:

$$\sigma_t^2 = \omega + \rho(L)\, \sigma_{t-1}^2 + \theta(L)\, \xi_t^2, \qquad /5/$$

where $\rho(L)$ is also a polynomial in the lag operator. This is called a $\text{GARCH}\,(p, q)$ model, when the order of polynomial $\rho(L)$ is p and the order of the polynomial $\theta(L)$ is q. The most commonly used model in the GARCH class is the simple $\text{GARCH}\,(1, 1)$ which can be written as

$$\begin{aligned} \sigma_t^2 &= \omega + \rho\, \sigma_{t-1}^2 + \theta\, \xi_t^2 = \omega + (\rho + \theta)\sigma_{t-1}^2 + \theta\, (\xi_t^2 - \sigma_{t-1}^2) = \\ &= \omega + (\rho + \theta)\, \sigma_{t-1}^2 + \theta\, \sigma_{t-1}^2\, (\varepsilon_t^2 - 1). \end{aligned} \qquad /6/$$

The term $(\xi_t^2 - \sigma_{t-1}^2)$ in the second equality in /6/ has zero mean, conditional on time t–1 information, and can be thought of as the shock to volatility. The coefficient θ measures the extent to which a volatility shock today feeds through into the next period's volatility, while $(\rho + \theta)$ measures the rate at which this effect dies out over time. The third equality in /6/ rewrites the volatility shock as $\sigma_{t-1}^2\, (\varepsilon_t^2 - 1)$, the square of a standard normal variable less its mean, i.e. a demeaned χ^2 (1) random variable, multiplied by past volatility σ_{t-1}^2.

The $\text{GARCH}\,(1, 1)$ model can also be written in terms of its implications for squared innovations ξ_{t+1}^2. We have then

$$\xi_{t+1}^2 = \omega + (\rho + \theta)\xi_t^2 + (\xi_{t+1}^2 - \sigma_t^2) - \rho(\xi_t^2 - \sigma_{t-1}^2). \qquad /7/$$

This last representation makes it clear that the $\text{GARCH}\,(1, 1)$ model is an $\text{ARMA}\,(1, 1)$ model for squared innovations, but the standard $\text{ARMA}\,(1, 1)$ model has homoscedastic shocks, while in this model the shocks $(\xi_{t+1}^2 - \sigma_t^2)$ are themselves heteroscedastic.

In the GARCH (1, 1) model it is easy to construct multiperiod forecasts of volatility. When $\rho + \theta < 1$, the unconditional variance of ξ_{t+1}, or equivalently the unconditional expectation of σ_t^2, is $\omega/(1-\rho-\theta)$.

The GARCH (1,1) model with $\rho + \theta = 1$ has a unit autoregressive root so that today's volatility affects forecasts of volatility into the indefinite future. It is therefore known as an integrated GARCH, or IGARCH (1,1) model (*Engle* and *Bollerslev*; 1986).

THE DATA

In the model specification the daily closing prices of stocks traded at the Budapest Stock Exchange (BSE) for the period August 1998 to January 2000 (365 trading day) are used. Results are based on a sample containing 18 individual securities as well as the stock compound index (BUX). The stocks under investigation (their names and codes used in the analysis are shown in the columnar composition) were selected because of their high volume (nearly 90 percent of the trading volume of the BSE) and frequency of trading in the last years. In the investigated time horizont of the analysis – practically without any changes – these stocks formed the stock index. This confirms the suitability of the sample as an adequate representation of the Hungarian stock market.

The stocks under investigation and their codes used in the study

Code	Stock	Code	Stock
BCHEM	Borsodchem Rt.	NABI	NABI Rt.
DANUB	Danubius Rt.	OTP	OTP Bank Rt.
DEMASZ	Démász Rt.	PPLAST	Pannonplast Rt.
EGIS	Egis Rt.	PICK	Pick Szeged Rt.
FOTEX	Fotex Rt.	PGAZ	Primagáz Rt.
GRABO	Graboplast Rt.	RABA	RÁBA Rt.
IEB	Inter-Europa Bank Rt.	RICHTER	Richter Gedeon Rt.
MATAV	Matáv Rt.	TVK	TVK Rt.
MOL	MOL Rt.	ZALAKER	Zalakerámia Rt.

Returns used to estimate the parameters of the market model were computed in the usual way by the formula

$$R_{it} = \frac{P_{it} - P_{i,t-1}}{P_{i,t-1}} \times \frac{365}{d_{t,t-1}},$$

where P_{it} is the closing price of stock i on day t and $d_{t,t-1}$ denotes the real number of days between trading days $t-1$ and t. This transformation (mean and variance stabilization) results in mean and covariance stationarity and ergodicity of the return series to guarantee the validity of all the statistical tests containing as an assumption the stationarity of the time series under investigation. Return values computed by the previous formula approximate the log returns widely used in finance. The market return was determined by the changes of the stock index (BUX).

THE EFFICIENCY OF BETA ESTIMATES, NORMALITY AND HETEROSCEDASTICITY OF THE RESIDUAL TERMS

As a first step of the empirical analysis the usually specified model (see equation /1/, white noise with random error) was estimated using the method of ordinary least squares (OLS), then tested the normality and heteroscedasticity of the residual terms. The normality test was performed by the Jarque-Bera statistic. For testing the heteroscedasticity the White test (*White*; 1980) was used. The results are summarized in Table 1.

Table 1

Results of parameter estimation with the OLS technique

Stock	BETA* estimates	*p*-value	Jarque-Bera statistic	*p*-value	White statistic	*p*-value
BCHEM	1.103 (23.28)	0.000	154.84	0.000	16.90	0.000
DANUB	0.771 (15.06)	0.000	100.80	0.000	60.90	0.000
DEMASZ	0.784 (19.64)	0.000	514.36	0.000	51.60	0.000
EGIS	1.046 (18.05)	0.000	477.96	0.000	10.90	0.000
FOTEX	0.658 (12.09)	0.000	12 043.88	0.000	22.85	0.000
GRABO	0.852 (8.69)	0.000	12 476.17	0.000	12.09	0.000
IEB	0.474 (9.03)	0.000	438.06	0.000	2.28	0.104
MATAV	0.735 (27.39)	0.000	351.33	0.000	67.83	0.000
MOL	0.834 (28.69)	0.000	11.71	0.003	6.06	0.003
NABI	0,983 (14.8)	0.000	831.10	0.000	6.58	0.002
OTP	1.189 (33.11)	0.000	729.43	0.000	20.43	0.000
PANNONPLAST	0.847 (12.91)	0.000	630.22	0.000	3.89	0.021
PICK	0.928 (15.44)	0.000	16 940.09	0.000	6.35	0.002
PRIMAGAZ	1.034 (18.53)	0.000	189.26	0.000	27.41	0.000
RÁBA	0.963 (17.98)	0.000	543.77	0.000	12.26	0.000
RICHTER	1.522 (26.56)	0.000	3 266.85	0.000	2.39	0.093
TVK	1,124 (19.69)	0.000	1 467.73	0.000	3.48	0.032
ZALAKERÁMIA	1.074 (16.95)	0.000	728.84	0.000	5.95	0.003

* *t* statistics in parentheses.

In the second step of the analysis, the estimation procedure was repeated using a $\mathrm{GARCH}(1,1)$ model for the error term. Results are presented in Table 2. Table 3 contains the estimated parameters of the $\mathrm{GARCH}(1,1)$ model and the *p*-values for the deviations from zero.

Table 2

Estimates of betas, values of t-statistic, p-values, and the Jarque-Bera test values for individual securities using GARCH(1,1) model

Stock	BETA* estimates	*p*-value	Jarque-Bera statistic	*p*-value
BCHEM	1.069 (24.94)	0.000	131.69	0.000
DANUB	0.681 (13.56)	0.000	67.25	0.000
DEMASZ	0.693 (25.63)	0.000	164.51	0.000
EGIS	1.021 (23.63)	0.000	631.21	0,000
FOTEX	0.576 (24.99)	0.000	643.26	0.000
GRABO	0.654 (17.77)	0.000	1932.64	0.000
IEB	0.414 (11.95)	0.000	276.91	0.000
MATAV	0.717 (51.07)	0.000	307.38	0.000
MOL	0.833 (45.02)	0.000	6.42	0.040
NABI	0.672 (12.93)	0.000	92.66	0.000
OTP	1.150 (57.09)	0.000	88.89	0.000
PPLAST	0.874 (16.16)	0.000	375.02	0.000
PICK	0.678 (16.13)	0.000	528.00	0.000
PGAZ	1.026 (30.87)	0.000	169.33	0.000
RABA	0.931 (37.42)	0.000	291.82	0.000
RICHTER	1.349 (44.51)	0.000	576.10	0.000
TVK	1.083 (19.75)	0.000	2226.53	0.000
ZALAKER	0.823 (25.17)	0.000	1380.39	0.000

* *t* statistics in parentheses.

Table 3

The estimated parameters of the GARCH (1,1) model and the p-values for the deviances from zero

Stock	$\hat{\theta}$	*p*-value	$\hat{\rho}$	*p*-value	$\hat{\theta}+\hat{\rho}$
BCHEM	0.082	0.000	0.870	0.000	0.952
DANUB	0.049	0.011	0.916	0.000	0.965
DEMASZ	0.084	0.000	0.860	0.000	0.944
EGIS	0.191	0.012	0.509	0.008	0.700
FOTEX	1.074	0.000	0.303	0.000	1.377
GRABO	0.248	0.000	0.691	0.000	0.939
IEB	0.328	0.000	0.339	0.001	0.667
MATAV	-0.038	0.000	0.119	0.847	0.081
MOL	0.141	0.010	0.673	0.000	0.814
NABI	0.077	0.000	0.910	0.000	0.987
OTP	0.175	0.000	0.757	0.000	0.932
PPLAST	0.411	0.000	0.311	0.000	0.722
PICK	0.239	0.000	0.576	0.000	0.815
PGAZ	0.131	0.000	0.284	0.183	0.415
RABA	0.336	0.000	0.451	0.000	0.787
RICHTER	0.191	0.000	0.793	0.000	0.984
TVK	0.058	0.000	0.837	0.000	0.895
ZALAKER	0.468	0.000	0.296	0.000	0.764

Evaluating the results the following can be stated.

1. The beta estimates based on both the OLS technique and the GARCH(1,1) adjusted model are the same from the view point of the risk evaluation with only one exception (ZALAKER) , being the estimated beta in the first case greater, and in the second case less than 1.

2. Based on the OLS estimates, it seems to be clear that in most of the models (16 out of 18) significant heteroscedasticity does exist.

3. Assuming the GARCH(1,1) model for the error term the estimates result in higher *t*-values than the ordinary method in 16 (out of 18) cases. (It should be noticed that all the beta estimates using even the ordinary or the GARCH(1,1) adjusted models are significantly different from zero.) It also should be emphasized that even in the case of GARCH specification the normality assumption of the residual variable does not hold, i. e., the increasing in *t*-values does not necessarily mean significant improvement.

4. The results of the normality test (Jarque-Bera test) also represent an improvement (in 15 out of 18 cases the test statistics are lower), but the residuals are not normally distributed.

5. The estimates of parameter α in the market model tends to be zero indicating the efficiency of the security market, because in an efficient market assets tend to flow to higher return securities or portfolios.

Calculations were repeated for portfolios in order to test whether some differences arose from grouping of individual stocks. Different groups of stocks were composed to test the influence of the size and composition of portfolios.

The investigation was conducted for three portfolios with different size and composition. Composition was determined on the basis of the stock's weights in the stock index. The weights are proportional to the size of capitalization. The portfolios under investigation are as follows:

1. PORT1: Consists of the most traded stocks (nearly half of the trading volume); MATAV, MOL, OTP, RICHTER, with the weights 1/4 all.

2. PORT2: A chemical industry's portfolio; BCHEM, GRABO, PPLAST, RICHTER, TVK, with proportions of 15-10-3-6-48-18 percents, respectively.

3. PORT3: A power industry's portfolio; DEMASZ, MOL, PGAZ, with weights of 30-60-10 percents, respectively.

Table 4

Results of the portfolio analysis using the OLS technique

Portfolios	BETA* estimates	*p*-value	Jarque-Bera statistic	*p*-value	White statistic	*p*-value
PORT1	1.070 (68.94)	0.000	639.89	0.000	7.51	0.001
PORT2	1.290 (42.7)	0.000	1072.71	0.000	8.47	0.000
PORT3	0.820 (36.82)	0.000	100.23	0.000	4.64	0.010

* *t* statistics in parentheses.

Table 5

Results of the portfolio analysis using GARCH (1,1) model for the error term

Portfolios	Estimated BETA*	*p*-value	Jarque-Bera statistic	*p*-value
PORT1	1.094 (71.71)	0.000	1330.69	0.000
PORT2	1.261 (106.41)	0.000	293.07	0.000
PORT3	0.817 (51.52)	0.000	71.94	0.000

* *t* statistics in parentheses.

Table 6

Parameter estimates of the portfolios with the GARCH (1,1) adjusted model

Portfolios	$\hat{\theta}$	*p*-value	$\hat{\rho}$	*p*-value	$\hat{\rho}+\hat{\theta}$
PORT1	0.109	0.000	0.848	0.000	0.957
PORT2	0.282	0.000	0.620	0.000	0.902
PORT3	0.169	0.003	-0.236	0.326	-0.067

These portfolios are different with respect to the number of individual securities composing the portfolios, as well as the individual securities in the compositions. Tables 4, 5, and 6 present the result of the analysis. As it can be seen, the findings for portfolios are keeping with those of individual securities. The estimates become more efficient in all the cases with minimum changes in betas, however normality does not hold.

CONCLUSION

The present paper emphasizes the importance of the conditional heteroscedasticity in the market model residual terms. Non-normality and heteroscedasticity of those residual terms make the estimators inefficient and some significance tests invalid. Thus, it is necessary to take this matter into account in beta estimates, so that they become more accurate and reliable. There must also be pointed out that the results achieved for Hungarian stocks are similar to those of *Bera–Bubnys–Park* (1988) using the data of the United States stock market suggesting that the presence of conditional heteroscedasticity is a general problem in the market model on capital markets. In applications of the market model, as well as the more general CAPM, non-normality does not cause problems, because normality is a sufficient and not necessary condition for the theoretical model. It should be emphasized that non-normality confuses the validation of significance hypothesis tests for parameters.

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