Threshold spatial vector autoregressive with metric exogenous variables (TSpVARX) for regional inflation and money outflow prediction

Gama Putra Danu Sohibien
Statistics Indonesia-Badan Pusat Statistik,
Politeknik Statistika STIS,
Indonesia
Email: gamaputra@stis.ac.id

Setiawan Setiawan
(corresponding author)
Department of Statistics,
Institut Teknologi Sepuluh Nopember,
Indonesia
Email: setiawan@statistika.its.ac.id

Dedy Dwi Prastyo
Department of Statistics,
Institut Teknologi Sepuluh Nopember,
Indonesia,
Email: dedy-dp@statistika.its.ac.id

Currently, some time series models are formed to accommodate several aspects: the reciprocal relationship between variables, the influence of exogenous variables, spatial relationships, and nonlinear relationships between variables. Threshold vector autoregressive with exogenous variables (E-TVAR) is formed to overcome the reciprocal relationship between variables, the influence of exogenous variables, and the nonlinear relationship between variables (Tsagkanos et al. 2018). Spatial vector autoregressive (SpVAR) of Beenstock–Felsenstein (2019) can capture the phenomenon of the relationship between endogenous variables and spatial influence. However, these models cannot consider all four aspects simultaneously. Some economic variables, such as inflation and money outflow, are reciprocally related, influenced by metric exogenous variables, interrelated between regions, and have a nonlinear relationship (Islam−Ahmed 2023, Hendayanti et al. 2017, Yuhan−Sohibien 2018, Suhartono et al. 2018).

Therefore, this study aims to propose the TSpVARX that can contain all four of these simultaneously. We conduct a theoretical study to prove the consistent and asymptotically normal properties of the maximum likelihood estimation (MLE) estimator in the *g*-th regime of the TSpVARX model. In addition, we also conduct a simulation study with some scenarios to evaluate the performance of the MLE estimator in the TSpVARX model. After we conduct theoretical studies and simulations, the TSpVARX model is applied to predict the inflation and money outflow of Yogyakarta, Solo, and Semarang. Our study shows that TSpVARX is better than SpVARX in predicting the inflation of those three cities and the money outflow of Semarang based on the root mean square error (RMSE) and and symmetric mean absolute percentage error (SMAPE).

Keywords:
spatiotemporal,
nonlinear time series,
forecasting,
money outflow,
SpVAR,
threshold

Introduction

We often use time series modeling for predicting economic data. autoregressive integrated moving average (ARIMA), developed by Box and Jenkins in 1976, is a univariate model in which the prediction of endogenous variables depends on the value of endogenous variables in previous periods (dan Hibon 1997). This model was developed into ARIMAX to catch the impact of exogenous variables (Liu 1980, Wei 2006, Cryer−Chan 2008). ARIMAX cannot involve the second, third, and fourth facets.

Sims (1977) states that all economic variables move together. It triggers the development of vector autoregressive (VAR) (Sims 1977, Hannan 1970). In VAR, predicting an endogenous variable depends on the movement of the variable itself and other variables in previous periods. VAR develops into VARX that can consider the impact of exogenous variables (Sukono et al. 2023, Prastuti et al. 2022, Apriliadara et al. 2016, Sohibien et al. 2022). Granger–Terasvirta (1993) state that the relationship between economic variables is usually nonlinear. Enders (2004) states that many economic variables show nonlinear behaviour. This aspect triggered the development of the threshold autoregressive (TAR) model introduced by Tong (1983) and Tsay (1989). TAR considers nonlinear relationships between variables by dividing the autoregressive model into separate regimes. The TAR model is modified into TVAR to involve the reciprocal relationship between variables and the nonlinear relationship between variables (Zhou–Chen 2023, Yuhan–Sohibien 2018). Furthermore, TVAR is altered into E-TVAR to take into account the influence of exogenous variables (Tsagkanos et al. 2018).

Tobler (1979) in Anselin (1988) state that everything is related to everything else, but near things are more connected than distant things. Generalized space-time autoregressive (GSTAR) is a model that can capture spatial relationships for one type of time series variable (Imro'ah 2023, Ruchjana et al. 2012). GSTAR transforms into

GSTARX to involve the influence of exogenous variables (Suhartono et al. 2018, Monica et al. 2021). However, GSTARX has the weakness of being unable to adjust the reciprocal relationship between variables. Therefore, it becomes an idea for spatial vector autoregressive (SpVAR) development. This model can catch spatial relationships for multiple time series variables (Beenstock–Felsenstein 2019). SpVAR is updated again to SpVAR with calendar variation (Sumarminingsih et al. 2018). However, SpVAR cannot catch the nonlinear relationship between variables and the influence of metric exogenous variables.

Based on the previous explanation, we can see that all previous studies have developed models to involve the aspects needed in time series modeling. However, based on the search results, we have not found a time series model that can simultaneously capture the aspects of the reciprocal relationship between variables, the influence of exogenous variables, the spatial aspects, and the nonlinearity of the relationship between variables. Therefore, we propose TSpVARX, which can adjust those facets simultaneously. We will estimate the TSpVARX's coefficient using MLE. We use TSpVARX to predict inflation and money outflow of Yogyakarta, Solo, and Semarang.

We apply TSpVARX to predict inflation and money outflow in Semarang, Solo, and Yogyakarta. We use money outflow as an approximation of the local money supply because the local money supply is unavailable. There are some reasons why we apply TSpVARX to inflation and money outflow predictions. Inflation as a price-raising indicator is vital. It can harm other macroeconomic variables if not controlled. Thus, local governments need inflation prediction as an early warning tool for future inflation. In addition, inflation and money outflow modeling allows the involvement of the previously described aspects, namely the interrelationship between variables, the impact of exogenous variables, the spatial aspect, and the nonlinear relationship between endogenous variables.

Some studies have found a two-way causality relationship between inflation and money outflow (Sumarminingsih et al. 2018, Denbel et al. 2016, Islam–Ahmed 2023). From the side of exogenous variables impact, Bank Indonesia's benchmark interest rate and the rupiah exchange rate against the dollar impact inflation and money outflow. When inflation is high enough, Bank Indonesia will raise interest rates to reduce inflation (Abdullah–Wahjusaputri 2018). Meanwhile, the effect of the rupiah exchange rate on the money outflow was explained by Hendayanti et al. (2017). When the rupiah weakens, people will be triggered to exchange dollars, thus increasing money outflow.

In terms of the relationship between locations, the movement of inflation in one location can also be related to other locations, which is called a spatial relationship. Commodities used to fulfill community demand do not only come from regional production but some goods are supplied from surrounding areas. Some studies that involve spatial aspects in modeling inflation and money outflow include Sumarminingsih et al. (2018), Suhartono et al. (2018), and Suhartono et al. (2016).

The next aspect that needs to be considered in modeling inflation and money outflow (as a local money supply approximation) is the possibility of a nonlinear relationship between variables. Many researchers have modeled inflation and money supply in Indonesia by incorporating aspects of nonlinearity. Yuhan–Sohibien (2018) examined the relationship between inflation, exchange rates, and money supply in Indonesia using TVAR and found that these variables can be modeled into two model regimes. Christopoulos et al. (2023) examined the impact of interest rate shocks on output and inflation using TVAR. They found that interest rates affect output and inflation when 'expected' inflation is more than 3.6%.

Based on the descriptions above, some objectives of our study are to develop a model and coefficient estimation of TSpVARX, to evaluate TSpVARX's performance compared to SpVARX, to predict inflation and money outflow of Semarang, Solo, and Yogyakarta from March–December 2023. We use Bank Indonesia's benchmark interest rate and the rupiah exchange rate against the US dollar as exogenous metric variables. The research areas we use are Yogyakarta, Solo, and Semarang. These three cities are used as loci because these three cities are geographically proximity and have easy access to interconnect. Besides that, Bank Indonesia representative office of these three cities can assist each other in the money supply. In addition, these three cities are known as the golden triangle area because they are the centre of economic development in Central Java, both from the tourism and industrial sectors (Kppip 2021). It allows for linkages in terms of economic activity between these three cities.

Method

Model and coefficient estimation of SpVARX

SpVARX is SpVAR that involves the impact of exogenous metric variables on endogenous variables. We can write SpVAR with a spatial lag order of one and a temporal lag order of *p* as SpVAR(1,*p*). The general form of SpVAR (1,*p*) can be written as follows (Di Giacinto 2010):

$$\begin{aligned} Y^1_{1,t} &= \omega^1_{10} + \gamma^{1,(1,0)}_{11} Y^1_{1,t-1} + \gamma^{1,(1,1)}_{11}\left(w_{11(1,2)}Y^2_{1,t-1} + \cdots + w_{11(1,u)}Y^u_{1,t-1} + \cdots + w_{11(1,N)}Y^N_{1,t-1}\right) + \cdots + \gamma^{1,(j,0)}_{1r} Y^1_{r,t-j} \\ &+ \gamma^{1,(j,1)}_{1r}\left(w_{1r(1,2)}Y^2_{r,t-j} + \cdots + w_{1r(1,u)}Y^u_{r,t-j} + \cdots + w_{1r(1,N)}Y^N_{r,t-j}\right) + \cdots + \gamma^{1,(p,0)}_{1K} Y^1_{K,t-p} \\ &+ \gamma^{1,(p,1)}_{1K}\left(w_{1K(1,2)}Y^2_{K,t-p} + \cdots + w_{1K(1,u)}Y^u_{K,t-p} + \cdots + w_{1K(1,N)}Y^N_{K,t-p}\right) + e^1_{1t}; \end{aligned} \tag{1a}$$

$$\begin{aligned} Y^2_{1t} &= \omega^2_{10} + \gamma^{2,(1,0)}_{11} Y^2_{1,t-1} + \gamma^{2,(1,1)}_{11}\left(w_{11(2,1)}Y^1_{1,t-1} + \cdots + w_{11(2,u)}Y^u_{1,t-1} + \cdots + w_{11(2,N)}Y^N_{1,t-1}\right) + \cdots + \gamma^{2,(j,0)}_{1r} Y^2_{r,t-j} \\ &+ \gamma^{2,(j,1)}_{1r}\left(w_{1r(2,1)}Y^1_{r,t-j} + \cdots + w_{1r(2,u)}Y^u_{r,t-j} + \cdots + w_{1r(2,N)}Y^N_{r,t-j}\right) + \cdots + \gamma^{2,(p,0)}_{1K} Y^2_{K,t-p} \\ &+ \gamma^{2,(p,1)}_{1K}\left(w_{1K(2,1)}Y^1_{K,t-p} + \cdots + w_{1K(2,u)}Y^u_{K,t-p} + \cdots + w_{1K(2,N)}Y^N_{K,t-p}\right) + e^2_{1t}, \end{aligned} \tag{1b}$$

$$\vdots$$

$$\begin{aligned} Y_{kt}^{n} = {} & \omega_{k0}^{n} + \gamma_{k1}^{n,(1,0)} Y_{1,t-1}^{n} + \gamma_{k1}^{n,(1,1)} \left(w_{k1(n,1)} Y_{1,t-1}^{1} + \cdots + w_{k1(n,n-1)} Y_{1,t-1}^{n-1} + w_{k1(n,n+1)} Y_{1,t-1}^{n+1} + \cdots + w_{k1(n,N)} Y_{1,t-1}^{N} \right) + \cdots + \gamma_{kr}^{n,(j,0)} Y_{r,t-j}^{n} \\ & + \gamma_{kr}^{n,(j,1)} \left(w_{kr(n,1)} Y_{1,t-j}^{1} + \cdots + w_{kr(n,n-1)} Y_{r,t-1}^{n-1} + w_{kr(n,n+1)} Y_{r,t-1}^{n+1} + \cdots + w_{kr(n,N)} Y_{r,t-j}^{N} \right) + \cdots \\ & + \gamma_{kK}^{n,(p,0)} Y_{K,t-p}^{n} + \gamma_{kK}^{n,(p,1)} (w_{kK(n,1)} Y_{K,t-p}^{n} + \cdots + w_{kK(n,n-1)} Y_{K,t-1}^{n-1} + w_{kK(n,n+1)} Y_{K,t-1}^{n+1} + \cdots + w_{1K(n,N)} Y_{K,t-p}^{N}) + e_{kt}^{n} \; ; \end{aligned} \quad (1c)$$

$$\vdots$$

$$\begin{aligned} Y_{Kt}^{N} = {} & \omega_{K0}^{N} + \gamma_{K1}^{N,(1,0)} Y_{1,t-1}^{N} + \gamma_{K1}^{N,(1,1)} \left(w_{K1(N,1)} Y_{1,t-1}^{1} + \cdots + w_{K1(N,u)} Y_{1,t-1}^{u} + \cdots + w_{K1(N,N-1)} Y_{1,t-1}^{N-1} \right) + \cdots + \gamma_{Kr}^{N,(j,0)} Y_{r,t-j}^{N} \\ & + \gamma_{Kr}^{N,(j,1)} \left(w_{Kr(N,1)} Y_{r,t-j}^{1} + \cdots + w_{Kr(N,u)} Y_{r,t-j}^{u} + \cdots + w_{Kr1(N,n)} Y_{r,t-j}^{N-1} \right) + \cdots \\ & + \gamma_{KK}^{N,(p,0)} Y_{K,t-p}^{N} + \gamma_{KK}^{N,(p,1)} (w_{KK(N,1)} Y_{K,t-p}^{1} + \cdots + w_{KK(N,u)} Y_{K,t-p}^{u} + \cdots + w_{KK(N,N-1)} Y_{K,t-p}^{N-1}) + e_{Kt}^{N} \; , \end{aligned} \quad (1d)$$

where: N is the number of locations; K is several endogenous variables; j is the notation for the lag of endogenous variables that is from 1 until p; $Y_{k,t}^{n}$ is the k-th endogenous variable in the n-th location at the t-th period; $Y_{r,t-j}^{u}$ is the predetermined variable in the form of the r-th endogenous variable with the lag of j and location of u; α_{k0}^{n} is the SpVAR intercept coefficient for the n-th location equation and the k-th endogenous variable; $\gamma_{kr}^{n,(j,0)}$ is autoregressive coefficients of predetermined variables $Y_{r,t-j}^{n}$ in equation Y_{kt}^{n} ; $\gamma_{kr}^{n,(j,1)}$ is the space-time coefficient for $w_{kr(n,1)} Y_{1,t-j}^{1} + \cdots + w_{kr(n,n-1)} Y_{r,t-1}^{n-1} + w_{kr(n,n+1)} Y_{r,t-1}^{n+1} + \cdots + w_{kr(n,N)} Y_{r,t-j}^{N}$ in equation Y_{kt}^{n} ; $w_{kr(n,N)}$ is the spatial weight between Y_{kt}^{n} and $Y_{r,t-j}^{N}$, with $j = 1, 2, \ldots, p$; e_{kt}^{n} is an error model of SpVARX for the equation of n-th location and the k-th endogenous variable.

We can form SpVARX by combining exogenous metric variables as predetermined variables into the SpVAR. The m-th exogenous variable with the (t–i)-th period and the n-th location is denoted by $F_{m,t-i}^{n}$ with $\lambda_{km}^{n,(i)}$ as the coefficient. The SpVARX with spatial lag order of one, temporal lag order of p, and exogenous variable lag order of q can be written as SpVARX (1, p, q). The general form of the SpVARX (*1*, p, q) model is as follows:

$$\begin{aligned} Y_{1,t}^{1} = {} & \omega^{1}{}_{10} + \lambda_{11}^{1,(1)} F_{1,t-1}^{1} + \cdots + \lambda_{11}^{1,(q)} F_{1,t-q}^{1} + \cdots + \lambda_{1m}^{1,(i)} F_{m,t-1}^{1} + \cdots + \gamma_{1M}^{1,(1)} F_{M,t-1}^{1} + \cdots + \lambda_{1M}^{1,(q)} F_{M.t-q}^{1} + \gamma_{11}^{1,(1,0)} Y^{1}{}_{1,t-1} \\ & + \gamma_{11}^{1,(1,1)} \left(w_{11(1,2)} Y_{1,t-1}^{2} + \cdots + w_{11(1,u)} Y_{1,t-1}^{u} + \cdots + w_{11(1,N)} Y_{1,t-1}^{N} \right) + \cdots + \gamma_{1r}^{1,(j,0)} Y^{1}{}_{r,t-j} \\ & + \gamma_{1r}^{1,(j,1)} \left(w_{1r(1,2)} Y_{r,t-j}^{2} + \cdots + w_{1r(1,u)} Y_{r,t-j}^{u} + \cdots + w_{1r(1,N)} Y_{r,t-j}^{N} \right) \\ & + \cdots + \gamma_{1K}^{1,(p,0)} Y_{K,t-p}^{1} + \gamma_{1K}^{1,(p,1)} \left(w_{1K(1,2)} Y_{K,t-p}^{2} + \cdots + w_{1K(1,u)} Y_{K,t-p}^{u} + \cdots + w_{1K(1,N)} Y_{K,t-p}^{N} \right) + e_{1t}^{1}, \end{aligned} \quad (2a)$$

$$
\begin{aligned}
Y_{1t}^{2} = {} & \omega_{10}^{2} + \lambda_{11}^{2,(0)} F_{1,t-1}^{2} + \cdots + \lambda_{11}^{2,(q)} F_{1,t-q}^{2} + \cdots + \lambda_{1m}^{2,(i)} F_{m,t-i}^{2} + \cdots + \lambda_{1M}^{2,(0)} F_{M,t-1}^{2} + \cdots + \lambda_{1M}^{2,(q)} F_{M,t-q}^{2} + \gamma_{11}^{2,(1,0)} Y_{1,t-1}^{2} \\
& + \gamma_{11}^{2,(1,1)} \left(w_{11(2,1)} Y_{1,t-1}^{1} + \cdots + w_{11(2,u)} Y_{1,t-1}^{u} + \cdots + w_{11(2,N)} Y_{1,t-1}^{N} \right) + \cdots + \gamma_{1r}^{2,(j,0)} Y_{r,t-j}^{2} \\
& + \gamma_{1r}^{2,(j,1)} \left(w_{1r(2,1)} Y_{r,t-j}^{1} + \cdots + w_{1r(2,u)} Y_{r,t-j}^{u} + \cdots + w_{1r(2,N)} Y_{r,t-j}^{N} \right) + \cdots + \gamma_{1K}^{2,(p,0)} Y_{K,t-p}^{2} \\
& + \gamma_{1K}^{2,(p,1)} \left(w_{1K(2,1)} Y_{K,t-p}^{1} + \cdots + w_{1K(2,u)} Y_{K,t-p}^{u} + \cdots + w_{1K(2,N)} Y_{K,t-p}^{N} \right) + e_{1t}^{2},
\end{aligned}
\tag{2b}
$$

$$\vdots$$

$$
\begin{aligned}
Y_{kt}^{n} = {} & \omega_{k0}^{n} + \lambda_{k1}^{n,(1)} F_{1,t-1}^{n} + \cdots + \lambda_{k1}^{n,(q)} F_{1,t-q}^{n} + \cdots + \lambda_{km}^{n,(i)} F_{m,t-i}^{n} + \cdots + \lambda_{kM}^{n,(1)} F_{M,t-1}^{n} + \cdots + \lambda_{kM}^{n,(q)} F_{M,t-q}^{n} + \gamma_{k1}^{n,(1,0)} Y_{1,t-1}^{n} \\
& + \gamma_{k1}^{n,(1,1)} \left(w_{k1(n,1)} Y_{1,t-1}^{1} + \cdots + w_{k1(n,n-1)} Y_{1,t-1}^{n-1} + w_{k1(n,n+1)} Y_{1,t-1}^{n+1} + \cdots + w_{k1(n,N)} Y_{1,t-1}^{N} \right) + \cdots + \gamma_{kr}^{n,(j,0)} Y_{r,t-j}^{n} \\
& + \gamma_{kr}^{n,(j,1)} \left(w_{kr(n,1)} Y_{1,t-j}^{1} + \cdots + w_{kr(n,n-1)} Y_{r,t-1}^{n-1} + w_{kr(n,n+1)} Y_{r,t-1}^{n+1} + \cdots + w_{kr(n,N)} Y_{r,t-j}^{N} \right) + \cdots \\
& + \gamma_{kK}^{n,(p,0)} Y_{K,t-p}^{n} + \gamma_{kK}^{n,(p,1)} \left(w_{kK(n,1)} Y_{K,t-p}^{n} + \cdots + w_{kK(n,n-1)} Y_{K,t-1}^{n-1} + w_{kK(n,n+1)} Y_{K,t-1}^{n+1} + \cdots + w_{1K(n,N)} Y_{K,t-p}^{N} \right) + e_{kt}^{n},
\end{aligned}
\tag{2c}
$$

$$\vdots$$

$$
\begin{aligned}
Y_{Kt}^{N} = {} & \omega_{K0}^{N} + \lambda_{K1}^{N,(1)} F_{1t-1}^{N} + \cdots + \lambda_{K1}^{N,(q)} F_{1,t-q}^{N} + \cdots + \lambda_{Km}^{N,(i)} F_{m,t-i}^{N} + \cdots + \lambda_{KM}^{N,(1)} F_{M,t-1}^{N} + \cdots + \lambda_{KM}^{N,(q)} F_{M,t-q}^{N} + \gamma_{K1}^{N,(1,0)} Y_{1,t-1}^{N} \\
& + \gamma_{K1}^{N,(1,1)} \left(w_{K1(N,1)} Y_{1,t-1}^{1} + \cdots + w_{K1(N,u)} Y_{1,t-1}^{u} + \cdots + w_{K1(N,N-1)} Y_{1,t-1}^{N-1} \right) + \cdots + \gamma_{Kr}^{N,(j,0)} Y_{r,t-j}^{N} \\
& + \gamma_{Kr}^{N,(j,1)} \left(w_{Kr(N,1)} Y_{r,t-j}^{1} + \cdots + w_{Kr(N,u)} Y_{r,t-j}^{u} + \cdots + w_{Kr(N,N-1)} Y_{r,t-j}^{N-1} \right) + \cdots \\
& + \gamma_{KK}^{N,(p,0)} Y_{K,t-p}^{N} + \gamma_{KK}^{N,(p,1)} \left(w_{KK(N,1)} Y_{K,t-p}^{1} + \cdots + w_{KK(N,u)} Y_{K,t-p}^{u} + \cdots + w_{KK(N,N-1)} Y_{K,t-p}^{N-1} \right) + e_{Kt}^{N},
\end{aligned}
\tag{2d}
$$

where: $F_{M,t-q}^{n}$ is M-th exogenous variable in n-th location at the (t-q)-th period; $\lambda_{kM}^{n,(q)}$ is the exogenous variable coefficient of $F_{M,t-q}^{n}$ in equation Y_{kt}^{n} ; M is the number of exogenous variables; i is the lag order notation of the exogenous variable, where i is from 1 until q.

We can also write equations (2a), (2b), (2c), and (2d) in the following form:

$$
\begin{bmatrix} Y_{1,t}^{1} \\ Y_{1,t}^{2} \\ \vdots \\ Y_{k,t}^{n} \\ \vdots \\ Y_{K,t}^{N} \end{bmatrix}
=
\begin{bmatrix}
(\mathbf{f}_{1,t}^{1})' & (\mathbf{y}_{1,t}^{1})' & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 \\
0 & 0 & (\mathbf{f}_{1,t}^{2})' & (\mathbf{y}_{1,t}^{2})' & \cdots & 0 & 0 & \cdots & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & 0 & 0 & \cdots & (\mathbf{f}_{k,t}^{n})' & (\mathbf{y}_{k,t}^{n})' & \cdots & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & (\mathbf{f}_{K,t}^{N})' & (\mathbf{y}_{K,t}^{N})'
\end{bmatrix}
\begin{bmatrix} \boldsymbol{\lambda}_{1}^{1} \\ \boldsymbol{\gamma}_{1}^{1} \\ \boldsymbol{\lambda}_{1}^{2} \\ \boldsymbol{\gamma}_{1}^{2} \\ \vdots \\ \boldsymbol{\lambda}_{k}^{n} \\ \boldsymbol{\gamma}_{k}^{n} \\ \vdots \\ \boldsymbol{\lambda}_{K}^{N} \\ \boldsymbol{\gamma}_{K}^{N} \end{bmatrix}
+
\begin{bmatrix} \mathrm{e}_{1,t}^{1} \\ \mathrm{e}_{1,t}^{2} \\ \vdots \\ \mathrm{e}_{k,t}^{n} \\ \vdots \\ \mathrm{e}_{K,t}^{N} \end{bmatrix}
\tag{3}
$$

where: $(\mathbf{f}_{k,t}^{n})' = \begin{bmatrix} 1 & F_{1,t-1}^{n} & \cdots & F_{1,t-q}^{n} & \cdots & F_{m,t-i}^{n} & \cdots & F_{M,t-1}^{n} & \cdots & F_{M,t-q}^{n} \end{bmatrix}$ with size 1 x (1+Mq); $(\mathbf{y}_{k,t}^{n})' = \begin{bmatrix} Y_{1,t-1}^{n} & Y_{k1,t-1}^{n*} & \cdots & Y_{r,t-j}^{n} & Y_{kr,t-j}^{n*} & \cdots & Y_{K,t-p}^{n} & Y_{kK,t-p}^{n*} \end{bmatrix}$ with size 1 x $2Kp$;

$Y_{kr,t-j}^{n*} = \sum_{u=1}^{N} w_{kr}(n,u) Y_{r,t-j}^{u}$; $\boldsymbol{\lambda}_k^n = \left[\omega_{k0}^n \quad \lambda_{k1}^{n(1)} \quad \cdots \quad \lambda_{k1}^{n(q)} \quad \cdots \quad \lambda_{km}^{n(i)} \quad \cdots \quad \lambda_{kM}^{n(1)} \quad \cdots \quad \lambda_{kM}^{n(q)}\right]'$ with size (1+*Mq*) x1; $\boldsymbol{\gamma}_k^n = \left[\gamma_{k1}^{n,(1,0)} \quad \gamma_{k1}^{n,(1,1)} \quad \cdots \quad \gamma_{kr}^{n,(j,0)} \quad \gamma_{kr}^{n,(j,1)} \quad \cdots \quad \gamma_{kK}^{n,(p,0)} \quad \gamma_{kK}^{n,(p,1)}\right]'$ with size 2*Kp* x 1; $\mathbf{e}_t = \left[e_{1,t}^1 \quad e_{1,t}^2 \quad \cdots \quad e_{k,t}^n \quad \cdots \quad e_{K,t}^N\right]'$ with size *NK* x 1.

Let $\mathbf{y}_t = \left[Y_{1,t}^1 \quad Y_{1,t}^2 \quad \cdots \quad Y_{k,t}^n \quad \cdots \quad Y_{K,t}^N\right]'$ with size *NK* x 1, $\mathbf{Y} = \begin{bmatrix} \mathbf{y}'_{h+1} \\ \mathbf{y}'_{h+2} \\ \vdots \\ \mathbf{y}'_{T} \end{bmatrix}$,

$\mathbf{F}_k^n = \begin{bmatrix} (\mathbf{f}_{k,h+1}^n)' & (\mathbf{y}_{k,h+1}^n)' \\ (\mathbf{f}_{k,h+2}^n)' & (\mathbf{y}_{k,h+2}^n)' \\ \vdots & \vdots \\ (\mathbf{f}_{k,T}^n)' & (\mathbf{y}_{k,T}^n)' \end{bmatrix}$, $\mathbf{F} = \begin{bmatrix} \mathbf{F}_1^1 & \mathbf{0} & \cdots & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{F}_1^2 & \cdots & \mathbf{0} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{F}_k^n & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \cdots & \mathbf{F}_K^N \end{bmatrix}$, $\mathbf{b}_k^n = \begin{bmatrix} \boldsymbol{\lambda}_k^n \\ \boldsymbol{\gamma}_k^n \end{bmatrix}$,

$\mathbf{b} = \left[\mathbf{b}_1^1 \quad \mathbf{b}_1^2 \quad \cdots \quad \mathbf{b}_k^n \quad \cdots \quad \mathbf{b}_K^N\right]'$; $\mathbf{E} = \left[\mathbf{e}_1^1 \quad \mathbf{e}_1^2 \quad \cdots \quad \mathbf{e}_k^n \quad \cdots \quad \mathbf{e}_K^N\right]$, and

$\mathbf{e}_k^n = \left[e_{k,h+1}^n \quad e_{k,h+2}^n \quad \cdots \quad e_{k,T}^n\right]'$ then if the number of observations used for the model is *T* and *h*=max(*p*,*q*), the form of SpVARX in equation (3) can be described in equation (4) as follows:

$$\text{Vec}(\mathbf{Y}) = \mathbf{Fb} + \text{Vec}(\mathbf{E}) \tag{4}$$

where: Vec is the operator that stacks a matrix as a column vector; $\text{Vec}(\mathbf{Y})$ is a vector of size *NK (T-h)* x 1 obtained from stacking **Y**; $\text{Vec}(\mathbf{E})$ is a vector of size *NK (T-h)* x 1 obtained from stacking **E**; **F** is a matrix of size *NK*(*T-h*) x *NK*(1+*Mq*+2*Kp*); **b** is a vector of size *NK*(1+*Mq*+2*Kp*) x 1.

Some stages in estimating the coefficients of the SpVARX model are:

1. Forming $\text{Vec}(\mathbf{Y})$, **F,** and, **b**, as in equation 4,
2. Estimating the coefficients of the SpVARX model with Ordinary Least Square (OLS) using the following formula:

$$\hat{\mathbf{b}}_{OLS,SpVARX} = \left(\mathbf{F}'\mathbf{F}\right)^{-1} \mathbf{F}'\text{Vec}(\mathbf{Y}) \tag{5}$$

3. Finding the residual of the SpVARX model obtained with OLS in step 2 with the following formula:

$$\text{Vec}(\mathbf{E})_{OLS} = \text{Vec}(\mathbf{Y}) - \mathbf{F}\hat{\mathbf{b}}_{OLS,SpVARX} \tag{6}$$

4. Finding the covariance matrix estimator of the SpVARX model with the following formula:

$$\hat{\mathbf{\Sigma}}_{OLS,SpVARX} = \begin{bmatrix} Var(\mathbf{e}_1^1) & Cov(\mathbf{e}_1^1,\mathbf{e}_1^2) & \cdots & Cov(\mathbf{e}_1^1,\mathbf{e}_k^n) & \cdots & Cov(\mathbf{e}_1^1,\mathbf{e}_K^N) \\ Cov(\mathbf{e}_1^2,\mathbf{e}_1^1) & Var(\mathbf{e}_1^2) & \cdots & Cov(\mathbf{e}_1^2,\mathbf{e}_k^n) & \cdots & Cov(\mathbf{e}_1^2,\mathbf{e}_K^N) \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ Cov(\mathbf{e}_1^n,\mathbf{e}_k^1) & Cov(\mathbf{e}_2^n,\mathbf{e}_k^1) & \cdots & Var(\mathbf{e}_k^n) & \cdots & Cov(\mathbf{e}_2^n,\mathbf{e}_K^N) \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ Cov(\mathbf{e}_k^n,\mathbf{e}_k^1) & Cov(\mathbf{e}_k^n,\mathbf{e}_k^1) & \cdots & Cov(\mathbf{e}_k^n,\mathbf{e}_k^1) & \cdots & Var(\mathbf{e}_K^N) \end{bmatrix}$$

where:

$$Var(\mathbf{e}_k^n) = \frac{1}{(T-h)}(\mathbf{e}_k^n)' . (\mathbf{e}_k^n)$$

$$Cov(\mathbf{e}_k^n,\mathbf{e}_K^N,) = \frac{1}{(T-h)}(\mathbf{e}_k^n)' . (\mathbf{e}_K^N)$$

5. Perform coefficient estimation of the SpVARX model coefficients with Maximum Likelihood Estimation (MLE) using the following formula:

$$\hat{\mathbf{b}}_{MLE,SpVARX} = \left(\mathbf{F}'\left(\hat{\Sigma}_{OLS,SpVARX} \otimes \mathbf{I}\right)\mathbf{F}\right)^{-1} \mathbf{F}'\left(\hat{\Sigma}_{OLS,SpVARX} \otimes \mathbf{I}\right)^{-1} \mathrm{Vec}(\mathbf{Y}) \quad (7)$$

where: **I** is an identity matrix with size (*T-h*) x (*T-h*).

Model development and coefficient estimation of TSpVARX

TSpVARX is developed by dividing time series data into some regimes based on the selected threshold variable and the threshold value. Each regime has a SpVAR model with metric exogenous variables (SpVARX). The division of the model into these regimes is based on the threshold value $(\hat{\varsigma})$ obtained from the threshold variable. The lag of the threshold variable named delay (*d*) can be interpreted as the length of time series data to switch regimes. The threshold variable with the delay of *d* in TSpVARX is denoted by $Y_{k,t-d}^u$. TSpVARX consists of *s* regimes with a delay order of *d*, spatial order of one, the temporal lag order of *p*, and exogenous variable lag order of *q* can be written as TSpVARX (*s*, 1, *p*, *q*, *d*). By using the SpVARX form of equation (4), we can write TSpVARX with two regimes as follows:

$$\mathrm{Vec}(\mathbf{Y}) = \begin{cases} \mathbf{F}^{(1)}\mathbf{b}^{(1)} + \mathrm{Vec}(\mathbf{E})^{(1)}, & \text{when } Y_{k,t-d}^u \le \varsigma, \\ \mathbf{F}^{(2)}\mathbf{b}^{(2)} + \mathrm{Vec}(\mathbf{E})^{(2)}, & \text{when } Y_{k,t-d}^u > \varsigma, \end{cases} \quad (8)$$

where: ς is a threshold value.

By adopting Hansen (1999), Lo–Zivot (2001), and Sohibien et al. (2024), there are several steps to estimate the TSpVARX coefficient with a threshold variable $\left(Y_{k,t-d}^{u}\right)$.

1. We set an endogenous lag variable which will be the threshold variable $\left(Y_{k,t-d}^{u}\right)$.
2. We specify the order *p* you want to use based on the smallest Akaike information criterion (AIC) of the SpVAR model. The formula of AIC can be found in Wei (2006). In this study, the maximum *p* is limited to 4.
3. We specify the order *q* orders to be used based on the smallest AIC of the SpVARX model. In this study, the maximum *q* is limited to 4.
4. We set the largest delay limit (*d*) of the candidate threshold variable equal to order *p* so that the candidate threshold variable is $Y_{k,t-1}^{u}$, $Y_{k,t-2}^{u}$, …, $Y_{k,t-p}^{u}$.
5. For each candidate threshold variable $\left(Y_{k,t-d}^{u} \text{ for } d = 1,\ 2,\ ...,\ p\right)$, we specify ς_{dL} = 10-th percentile and ς_{dU} = 90-th percentile of $Y_{k,t-d}^{u}$.
6. We specify $\varsigma_{dL} \le \varsigma \le \varsigma_{dU}$.
7. We perform all possible data divided into two regimes based on all possible combinations (d,ς). The data is divided using the following ways:
   - If $Y_{k,t-d}^{u} \le \varsigma$ then the observation on the *t*-th period will fall into the first regime,
   - If $Y_{k,t-d}^{u} > \varsigma$ then observation on the *t*-th period will fall into the second regime.
8. We estimate the first and second regime coefficients of TSpVARX with two regimes using MLE for each possible data division. The estimation method is the same as the SpVARX estimation step described in the previous section.
9. We Calculate the value of the log-likelihood function in the first regime $l\left(\hat{\mathbf{b}}_{MLE,SpVARX}^{(1)}(d,\varsigma) \mid \mathbf{\Omega}_{OLS,SpVARX}^{(1)}\right)$ and the second regime $l\left(\hat{\mathbf{b}}_{MLE,SpVARX}^{(2)}(d,\varsigma) \mid \mathbf{\Omega}_{OLS,SpVARX}^{(2)}\right)$ for each possible data division by the following formula:
$$l\left(\hat{\mathbf{b}}_{MLE,SpVARX}^{(g)}(d,\varsigma) \mid \mathbf{\Omega}_{OLS,SpVARX}^{(g)}\right) = -\frac{NKT^{(g)}}{2}\ln 2\pi - \frac{1}{2}\ln\left|\mathbf{\Omega}_{OLS,SpVARX}^{(g)}\right| - \frac{1}{2}\left[\left(\mathrm{Vec}(\mathbf{Y})^{(g)} - \mathbf{F}^{(g)}\hat{\mathbf{b}}_{MLE,SpVARX}^{(g)}\right)'\left(\mathbf{\Omega}_{OLS,SpVARX}^{(g)}\right)^{-1}\left(\mathrm{Vec}(\mathbf{Y})^{(g)} - \mathbf{F}^{(g)}\hat{\mathbf{b}}_{MLE,SpVARX}^{(g)}\right)\right]$$
   where: $l\left(\hat{\mathbf{b}}_{MLE,SpVARX}^{(g)}(d,\varsigma) \mid \mathbf{\Omega}_{OLS,SpVARX}^{(g)}\right)$ is the the log-likelihood function value in *g*-th regimes of TSpVARX with two regimes; $\mathbf{\Omega}_{OLS,SpVARX}^{(g)}$ is the error covariance matrix in the *g*-th regime that is gotten from the OLS method with size *KN*(*T*-*h*) x *KN*(*T*-*h*). We can get $\mathbf{\Omega}_{OLS,SpVARX}^{(g)}$ by using $\mathbf{\Omega}_{OLS,SpVARX}^{(g)} = \hat{\Sigma}_{OLS,SpVARX} \otimes \mathbf{I}$.
10. We calculate the total value of the log-likelihood function $\left(l_{total}(d,\varsigma)\right)$ for each combination *d* and ς by the following formula:

$$l_{total}\left(d,\varsigma\right)=l\left(\hat{\mathbf{b}}^{(1)}_{MLE,SpVARX}\left(d,\varsigma\right)|\,\mathbf{\Omega}^{(1)}_{OLS,SpVARX}\right)+l\left(\hat{\mathbf{b}}^{(2)}_{MLE,SpVARX}\left(d,\varsigma\right)|\,\mathbf{\Omega}^{(2)}_{OLS,SpVARX}\right)$$

where: $l_{total}\left(d,\varsigma\right)$ is the total value of the log-likelihood

11. We get the delay estimation $\left(\hat{d}\right)$ and threshold value estimation $\left(\hat{\varsigma}\right)$ by selecting d and ς that can maximize $l_{total}\left(d,\varsigma\right)$. We can write it in mathematical form as follows:
$\left(\hat{d},\hat{\varsigma}\right)=\left\{\left(d,\varsigma\right),\max l_{total}\left(d,\varsigma\right)\right\}$.
12. We get the coefficient estimation of the TSpVARX with two regimes in the g-th regime by using $\hat{d}$ and $\hat{\varsigma}$. We can write it in mathematical form as follows:
$\hat{\mathbf{b}}^{(g)}_{MLE,TSpVARX,2rezim}=\hat{\mathbf{b}}^{(g)}_{MLE,SpVARX}\left(\hat{d},\hat{\varsigma}\right)$.

We create two flow charts to make it easier to understand the flow in estimating TSpVARX with two regimes. The first flow chart in Figure 1 illustrates the flow for determining the estimation of the temporal lag order $\left(\tilde{p}\right)$ and the exogenous variables' lag order $\left(\tilde{q}\right)$. The second flow chart in Figure 2 shows the flow for estimating the coefficients of TSpVARX with two regimes.

Figure 1

The flow chart of determination of temporal lag order $\left(\tilde{p}\right)$ and exogenous variables lag order $\left(\tilde{q}\right)$ estimation for TSpVARX model

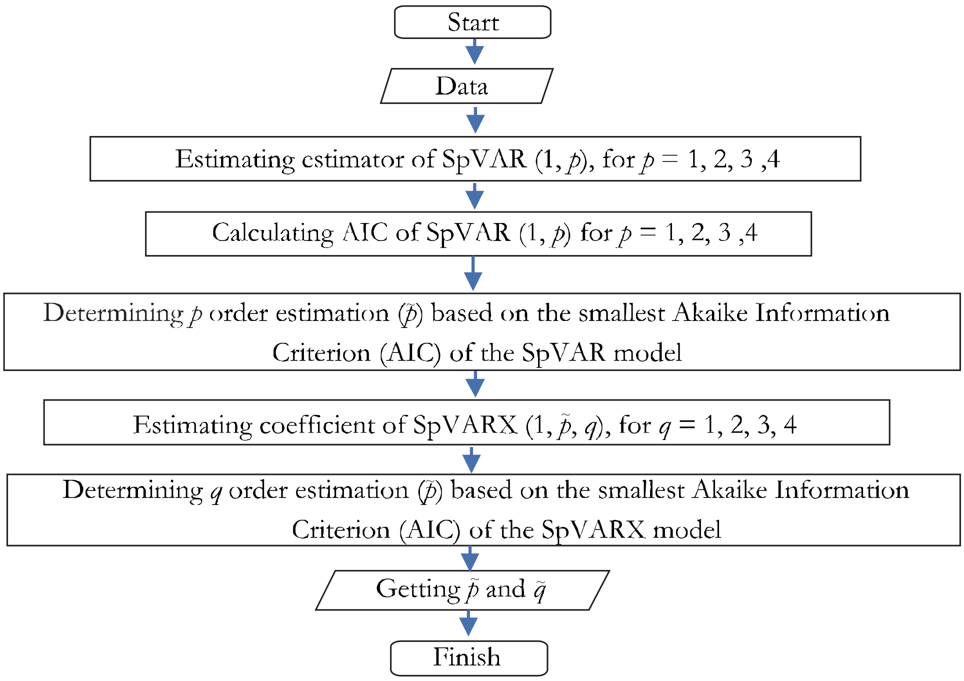
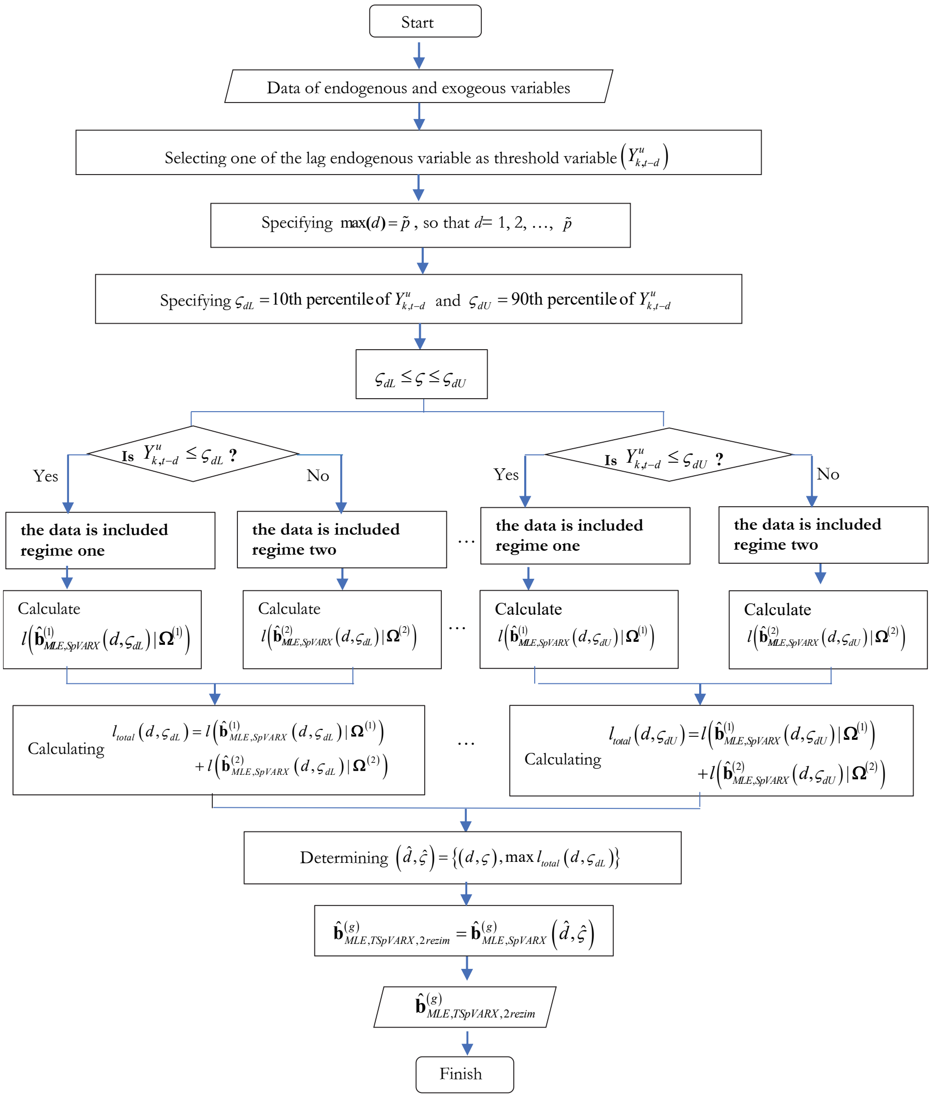


Figure 2
The flow chart of the coefficient estimation of TSpVARX with two regimes



Some of the properties of the coefficient estimators that are proven are consistent and asymptotically normal.

Theorem 1

If $\hat{\mathbf{b}}^{(g)}_{MLE,TSpVARX}$ is the coefficient estimator of the TSpVARX in the *g*-th regime using the MLE method, then $\hat{\mathbf{b}}^{(g)}_{MLE,TSpVARX}$ is a consistent estimator.

Theorem 2

If $\hat{\mathbf{b}}^{(g)}_{MLE,TSpVARX}$ is the coefficient estimation of the TSpVARX model in the *g*-th regime obtained using assumptions $\boldsymbol{\varepsilon}^{(g)} \sim N_{KN}\left(\mathbf{0},\boldsymbol{\Omega}^{(g)}\right)$ and $\boldsymbol{\Omega}^{(g)} = \boldsymbol{\Sigma}^{(g)}_{NK\times NK} \otimes \mathbf{I}_{T^{(g)}}$, $\hat{\mathbf{b}}^{(g)}_{MLE,TSpVARX}$ will be asymptotically normally distributed.

The proof of **theorem 1** and **2** can be seen in the Appendix.

Simulation study to evaluate the performance of the estimator

We conducted a simulation study to evaluate the performance of the MLE estimator on the TSpVARX model. The simulation is conducted using two endogenous variables, one metric exogenous variable, and three locations. This simulation is done by combining the error covariance conditions and the sample size conditions. Because there are two error covariance conditions and three sample size conditions used, there are six scenarios that will be used in this simulation. The conditions of the error covariance and sample size that will be used in this simulation are as follows:

1. There are two conditions of the error covariance matrix used to generate data, namely:
   a) the first condition of the covariance matrix is when the error correlation between equations is 0.1 (error covariance between endogenous variables is 0.01),
   b) the second condition of the covariance matrix is when the error correlation between equations is 0.9 (covariance error between endogenous variables is 0.09).
2. There are three sample size conditions used, namely 120, 240, and 360 samples.

There are some steps taken in conducting a simulation study of the TSpVARX model.

1. We generate normally distributed errors $(\in_t)$ with zero mean and 0.2 variance for as many simulation periods as possible to form simulated data of the exogenous variables.
2. We generate the exogenous variable data by using the following equation: $X_t = 0{,}5X_{t-1} + \in_t$, with $\in_t \sim N\left(0,\ 0{,}2\right)$.
3. We generate multivariate normally distributed errors $\left(\boldsymbol{\varepsilon} \sim N\left(\boldsymbol{\mu},(\Sigma\otimes \boldsymbol{I}_T)\right)\right)$ to form data simulations of endogenous variables in each regime for as many simulation periods as possible.

4. We generate endogenous variable data by using the following model:

The 1st regime (when $\boldsymbol{Y^1_{1,t-1} \le 0}$ **)**

$$Y^1_{1t} = 0.4X_{1,t-1} + 0.25Y^1_{1,t-1} + 0.3\left(0.5Y^2_{1,t-1} + 0.5Y^3_{1,t-1}\right) + 0.2Y^1_{2,t-1} + 0.5\left(0.5Y^2_{2,t-1} + 0.5Y^3_{2,t-1}\right) + e^1_1 ,$$

$$Y^2_1 = 0.55X_{1,t-1} + 0.2Y^2_{1,t-1} + 0.12\left(0.5Y^1_{1,t-1} + 0.5Y^3_{1,t-1}\right) + 0.3Y^2_{2,t-1} + 0.25\left(0.5Y^1_{2,t-1} + 0.5Y^3_{2,t-1}\right) + e^2_1 ,$$

$$Y^3_1 = 0.2X_{1,t-1} + 0.3Y^3_{1,t-1} + 0.3\left(0.5Y^1_{1,t-1} + 0.5Y^2_{1,t-1}\right) + 0.35Y^3_{2,t-1} + 0.4\left(0.5Y^1_{2,t-1} + 0.5Y^2_{2,t-1}\right) + e^3_1 ,$$

$$Y^1_2 = 0.5X_{1,t-1} + 0.15Y^1_{1,t-1} + 0.05\left(0.5Y^2_{1,t-1} + 0.5Y^3_{1,t-1}\right) + 0.125Y^1_{2,t-1} + 0.2\left(0.5Y^2_{2,t-1} + 0.5Y^3_{2,t-1}\right) + e^1_2 ,$$

$$Y^2_2 = 0.3X_{1,t-1} + 0.25Y^2_{1,t-1} + 0.3\left(0.5Y^1_{1,t-1} + 0.5Y^3_{1,t-1}\right) + 0.3Y^2_{2,t-1} + 0.05\left(0.5Y^1_{2,t-1} + 0.5Y^3_{2,t-1}\right) + e^2_2 ,$$

$$Y^3_{2,t} = 0.4X_{1,t-1} + 0.2Y^3_{1,t-1} + 0.3\left(0.5Y^1_{1,t-1} + 0.5Y^2_{1,t-1}\right) + 0.25Y^3_{2,t-1} + 0.02\left(0.5Y^1_{2,t-1} + 0.5Y^2_{2,t-1}\right) + e^3_2 ,$$

The 2nd regime (when $\boldsymbol{Y^1_{1,t-1} > 0}$ **)**

$$Y^1_{1t} = 0.2X_{1,t-1} + 0.2Y^1_{1,t-1} + 0.24\left(0.5Y^2_{1,t-1} + 0.5Y^3_{1,t-1}\right) + 0.16Y^1_{2,t-1} + 0.4\left(0.5Y^2_{2,t-1} + 0.5Y^3_{2,t-1}\right) + e^1_1 ,$$

$$Y^2_{1t} = 0.25X_{1,t-1} + 0.16Y^2_{1,t-1} + 0.096\left(0.5Y^1_{1,t-1} + 0.5Y^3_{1,t-1}\right) + 0.24Y^2_{2,t-1} + 0.2\left(0.5Y^1_{2,t-1} + 0.5Y^3_{2,t-1}\right) + e^2_1 ,$$

$$Y^3_1 = 0.1X_{1,t-1} + 0.24Y^3_{1,t-1} + 0.24\left(0.5Y^1_{1,t-1} + 0.5Y^2_{1,t-1}\right) + 0.28Y^3_{2,t-1} + 0.32\left(0.5Y^1_{2,t-1} + 0.5Y^2_{2,t-1}\right) + e^3_1 ,$$

$$Y^1_2 = 0.25X_{1,t-1} + 0.12Y^1_{1,t-1} + 0.04\left(0.5Y^2_{1,t-1} + 0.5Y^3_{1,t-1}\right) + 0.17Y^1_{2,t-1} + 0.016\left(0.5Y^2_{2,t-1} + 0.5Y^3_{2,t-1}\right) + e^2_1 ,$$

$$Y^2_2 = 0.1X_{1,t-1} + 0.2Y^2_{1,t-1} + 0.24\left(0.5Y^1_{1,t-1} + 0.5Y^3_{1,t-1}\right) + 0.24Y^2_{2,t-1} + 0.04\left(0.5Y^1_{2,t-1} + 0.5Y^3_{2,t-1}\right) + e^2_2 ,$$

$$Y^3_{2t} = 0.15X_{1,t-1} + 0.16Y^3_{1,t-1} + 0.24\left(0.5Y^1_{1,t-1} + 0.5Y^2_{1,t-1}\right) + 0.2Y^3_{2,t-1} + 0.016\left(0.5Y^1_{2,t-1} + 0.5Y^2_{2,t-1}\right) + e^3_2$$

5. We perform simulations with 100 repetitions (each repetition is performed as in steps 1 to 5).
6. We calculate the percentage of the sign of the coefficient estimators that are the same as the sign of the true coefficients of TSpVARX. The larger the percentage, the better the estimation of the model coefficients in terms of sign.
7. We calculate the percentage of TSpVARX parameter coefficients with MLE that are significant in the model. The larger the percentage, the better the estimated model coefficients are in terms of sign.

Dataset and variables for application

The data we used in this study are as follows:

1. Semarang's inflation, Solo's inflation, and Yogyakarta's inflation for the period January 2006–May 2023 (in percent),
2. Semarang's money outflow, Solo's money outflow, and Yogyakarta's money outflow for the period January 2006–December 2021 (in billion rupiahs),
3. The rupiah exchange rate against the dollar and Bank Indonesia's benchmark interest rate for January 2006–May 2023.

We get inflation data from Statistics Indonesia. Meanwhile, money outflow data, the rupiah exchange rate, and Bank Indonesia's benchmark interest rate are obtained from Bank Indonesia. Our endogenous variables are inflation of Semarang (Inf_Se), inflation of Solo (Inf_So), inflation of Yogyakarta (Inf_Yo), money outflow of Semarang (Out_Se), money outflow of Solo (Out_So), and money outflow of Yogyakarta (Out_Yo). Our exogenous metric variables are exchange rate depreciation (Dep) and Bank Indonesia's benchmark interest rate (BI_rate).

Formation stages of SpVARX and TSpVARX

There are some stages for getting the SpVARX and TSpVARX.

1. We test of data stationary using the Phillips–Perron (PP) test (Phillips−Perron 1988).
2. W identify the presence of spatial relationships between endogenous variables using cross-correlation values.
3. We find uniform spatial weight. We use it because Semarang, Solo, and Yogyakarta have equally easy access to connect in terms of transportation and economy. We can get uniform spatial weight by using the following formula:
$$w_{kr}(n,u) = \frac{1}{N_n},$$
where: $w_{kr}(n,u)$ is the weights of the *k*-th endogenous variable of the *n*-th location with the *r*-th endogenous variable of the *u*-th location; N_n is many locations that neighbour the *n*-th location; We find *p* order based on the minimum AIC of the SpVAR.
4. We find *q* order based on the minimum AIC of the SpVARX model with order *p* obtained from step 3.
5. We get the coefficient estimators of SpVARX (*p*,*q*).
6. We test the nonlinearity of the relationship between variables using Ramsey's reset test (see Gujarati 2003).
7. We can process TSpVARX modeling if there is a nonlinear relationship between endogenous variables and predetermined variables.
8. We specify the delay order (*d*) and threshold variable based on the generated AIC.
9. We estimate TSpVARX coefficients with selected orders *p*, *q*, and *d*.

Stages to evaluate predictive ability of SpVARX and TSpVARX

Some steps to evaluate the predictive ability of SpVARX and TSpVARX are:

1. Dividing data into two parts, namely in-sample data (January 2006–December 2020) and out-of-sample data (January–December 2021).

2. Modeling SpVARX and TSpVARX using in-sample data.
3. Predicting inflation and money outflow in the period of out-of-sample data using the SpVARX and TSpVARX obtained in the second stages.
4. Choosing the best model based on the smallest RMSE (Wei 2006).

Predicting procedure using TSpVARX in g-th regime

By adopting the predicting process on the ARIMA model described by Wei (2006), we predict endogenous variable values for future periods using the TSpVARX in the *g*-th regime recursively.

The prediction procedures for endogenous variables using TSpVARX in the *g*-th regime for the upcoming period are as follows:

1. Predicting procedure for one period ahead $(l = 1)$

$$\begin{bmatrix} \hat{Y}^1_{1,t+1} \\ \hat{Y}^2_{1,t+1} \\ \vdots \\ \hat{Y}^n_{k,t+1} \\ \vdots \\ \hat{Y}^N_{K,t+1} \end{bmatrix} = \begin{bmatrix} (\mathbf{f}^1_{1,t+1})' & (\mathbf{y}^1_{1,t+1})' & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & (\mathbf{f}^2_{1,t+1})' & (\mathbf{y}^2_{1,t+1})' & \cdots & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & (\mathbf{f}^n_{k,t+1})' & (\mathbf{y}^n_{k,t+1})' & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & (\mathbf{f}^N_{K,t+1})' & (\mathbf{y}^N_{K,t+1})' \end{bmatrix} \begin{bmatrix} (\boldsymbol{\lambda}^1_1)^{(g)} \\ (\boldsymbol{\gamma}^1_1)^{(g)} \\ (\boldsymbol{\lambda}^2_1)^{(g)} \\ (\boldsymbol{\gamma}^2_1)^{(g)} \\ \vdots \\ (\boldsymbol{\lambda}^n_k)^{(g)} \\ (\boldsymbol{\gamma}^n_k)^{(g)} \\ \vdots \\ (\boldsymbol{\lambda}^N_K)^{(g)} \\ (\boldsymbol{\gamma}^N_K)^{(g)} \end{bmatrix},$$

where:

$(\mathbf{f}^n_{k,t+1})' = [1 \;\; F^n_{1,t} \;\; \cdots \;\; F^n_{1,t+1-q} \;\; \cdots \;\; F^n_{m,t+1-i} \;\; \cdots \;\; F^n_{M,t} \;\; \cdots \;\; F^n_{M,t+1-q}]$ with size 1 x (1+*Mq*),

$(\mathbf{y}^n_{k,t+1})' = [Y^n_{1,t} \;\; Y^{n*}_{k1,t} \;\; \cdots \;\; Y^n_{r,t+1-j} \;\; Y^{n*}_{kr,t+1-j} \;\; \cdots \;\; Y^n_{K,t+1-p} \;\; Y^{n*}_{kK,t+1-p}]$ with size 1 x 2*KP*,

$Y^{n*}_{kr,t+1-j} = \sum_{u=1}^{N} w_{kr}(n,u) Y^u_{r,t+1-j}$,

$(\boldsymbol{\lambda}^n_k)^{(g)} = \left[(\omega^n_{k0})^{(g)} \;\; (\lambda^{n(1)}_{k1})^{(g)} \;\; \cdots \;\; (\lambda^{n(q)}_{k1})^{(g)} \;\; \cdots \;\; (\lambda^{n(i)}_{km})^{(g)} \;\; \cdots \;\; (\lambda^{n(1)}_{kM})^{(g)} \;\; \cdots \;\; (\lambda^{n(q)}_{kM})^{(g)}\right]'$ with size (1+*Mq*)x1

$(\boldsymbol{\gamma}^n_k)^{(g)} = \left[(\gamma^{n,(1,0)}_{k1})^{(g)} \;\; (\gamma^{n,(1,1)}_{k1})^{(g)} \;\; \cdots \;\; (\gamma^{n,(j,0)}_{kr})^{(g)} \;\; (\gamma^{n,(j,1)}_{kr})^{(g)} \;\; \cdots \;\; (\gamma^{n,(p,0)}_{kK})^{(g)} \;\; (\gamma^{n,(p,1)}_{kK})^{(g)}\right]'$ with size 2*KP* x 1,

2. Predicting procedure for two periods ahead $(l=2)$

$$\begin{bmatrix} \hat{Y}^1_{1,t+2} \\ \hat{Y}^2_{1,t+2} \\ \vdots \\ \hat{Y}^n_{k,t+2} \\ \vdots \\ \hat{Y}^N_{K,t+2} \end{bmatrix} = \begin{bmatrix} (\hat{\mathbf{f}}^1_{1,t+2})' & (\hat{\mathbf{y}}^1_{1,t+2})' & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & (\hat{\mathbf{f}}^2_{1,t+2})' & (\hat{\mathbf{y}}^2_{1,t+2})' & \cdots & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & (\hat{\mathbf{f}}^n_{k,t+2})' & (\hat{\mathbf{y}}^n_{k,t+2})' & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & (\hat{\mathbf{f}}^N_{K,t+2})' & (\hat{\mathbf{y}}^N_{K,t+2})' \end{bmatrix} \begin{bmatrix} \boldsymbol{\lambda}^1_1 \\ \boldsymbol{\gamma}^1_1 \\ \boldsymbol{\lambda}^2_1 \\ \boldsymbol{\gamma}^2_1 \\ \vdots \\ \boldsymbol{\lambda}^n_k \\ \boldsymbol{\gamma}^n_k \\ \vdots \\ \boldsymbol{\lambda}^N_K \\ \boldsymbol{\gamma}^N_K \end{bmatrix},$$

where: $(\hat{\mathbf{f}}^n_{k,t+2})' = \begin{bmatrix} 1 & \hat{F}^n_{1,t+1} & \cdots & \hat{F}^n_{1,t+2-q} & \cdots & \hat{F}^n_{m,t+2-i} & \cdots & \hat{F}^n_{M,t+1} & \cdots & \hat{F}^n_{M,t+2-q} \end{bmatrix}$ with size 1 x (1+Mq),

$(\hat{\mathbf{y}}^n_{k,t+2})' = \begin{bmatrix} \hat{Y}^n_{1,t+1} & \hat{Y}^{n*}_{k1,t+1} & \cdots & \hat{Y}^n_{r,t+2-j} & \hat{Y}^{n*}_{kr,t+2-j} & \cdots & \hat{Y}^n_{K,t+2-p} & \hat{Y}^{n*}_{kK,t+2-p} \end{bmatrix}$ with size 1 x 2KP,

$\hat{Y}^{n*}_{kr,t+2-j} = \sum_{u=1}^{N} w_{kr}(n,u)\hat{Y}^u_{r,t+2-j}$,

Do the next forecasting until the next L period forecast.

3. Predicting procedure for L period ahead $(l=L)$

$$\begin{bmatrix} \hat{Y}^1_{1,t+L} \\ \hat{Y}^2_{1,t+L} \\ \vdots \\ \hat{Y}^n_{k,t+L} \\ \vdots \\ \hat{Y}^N_{K,t+L} \end{bmatrix} = \begin{bmatrix} (\hat{\mathbf{f}}^1_{1,t+L})' & (\hat{\mathbf{y}}^1_{1,t+L})' & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & (\hat{\mathbf{f}}^2_{1,t+L})' & (\hat{\mathbf{y}}^2_{1,t+L})' & \cdots & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & (\hat{\mathbf{f}}^n_{k,t+L})' & (\hat{\mathbf{y}}^n_{k,t+L})' & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & (\hat{\mathbf{f}}^N_{K,t+L})' & (\hat{\mathbf{y}}^N_{K,t+L})' \end{bmatrix} \begin{bmatrix} \boldsymbol{\lambda}^1_1 \\ \boldsymbol{\gamma}^1_1 \\ \boldsymbol{\lambda}^2_1 \\ \boldsymbol{\gamma}^2_1 \\ \vdots \\ \boldsymbol{\lambda}^n_k \\ \boldsymbol{\gamma}^n_k \\ \vdots \\ \boldsymbol{\lambda}^N_K \\ \boldsymbol{\gamma}^N_K \end{bmatrix},$$

where: $(\hat{\mathbf{f}}^n_{k,t+L})' = \begin{bmatrix} 1 & \hat{F}^n_{1,t+L-1} & \cdots & \hat{F}^n_{1,t+L-q} & \cdots & \hat{F}^n_{m,t+L-i} & \cdots & \hat{F}^n_{M,t+L-1} & \cdots & \hat{F}^n_{M,t+L-q} \end{bmatrix}$ with size 1 x (1+Mq),

$(\mathbf{y}^n_{k,t+L})' = \begin{bmatrix} \hat{Y}^n_{1,t+L-1} & \hat{Y}^{n*}_{k1,t+L-1} & \cdots & \hat{Y}^n_{r,t+L-j} & \hat{Y}^{n*}_{kr,t+L-j} & \cdots & \hat{Y}^n_{K,t+L-p} & \hat{Y}^{n*}_{kK,t+L-p} \end{bmatrix}$ with size 1 x 2KP,

$\hat{Y}^{n*}_{kr,t+L-j} = \sum_{u=1}^{N} w_{kr}(n,u)\hat{Y}^u_{r,t+L-j}$, $Y^{n*}_{kr,t+L-j} = \sum_{u=1}^{N} w_{kr}(n,u)\hat{Y}^u_{r,t+L-j}$.

In forecasting two to L periods ahead, the estimated value of the m-th exogenous variable in the i-th lag $(\hat{F}^n_{m,t+2-i}, \hat{F}^n_{m,t+3-i}, ..., \hat{F}^n_{m,t+L-i})$ is obtained from the forecast results with the ARIMA. ARIMA's forecasting procedure can be seen in Wei (2006).

Stages to predict inflation and money outflow of Yogyakarta, Solo, and Semarang from June–December 2023

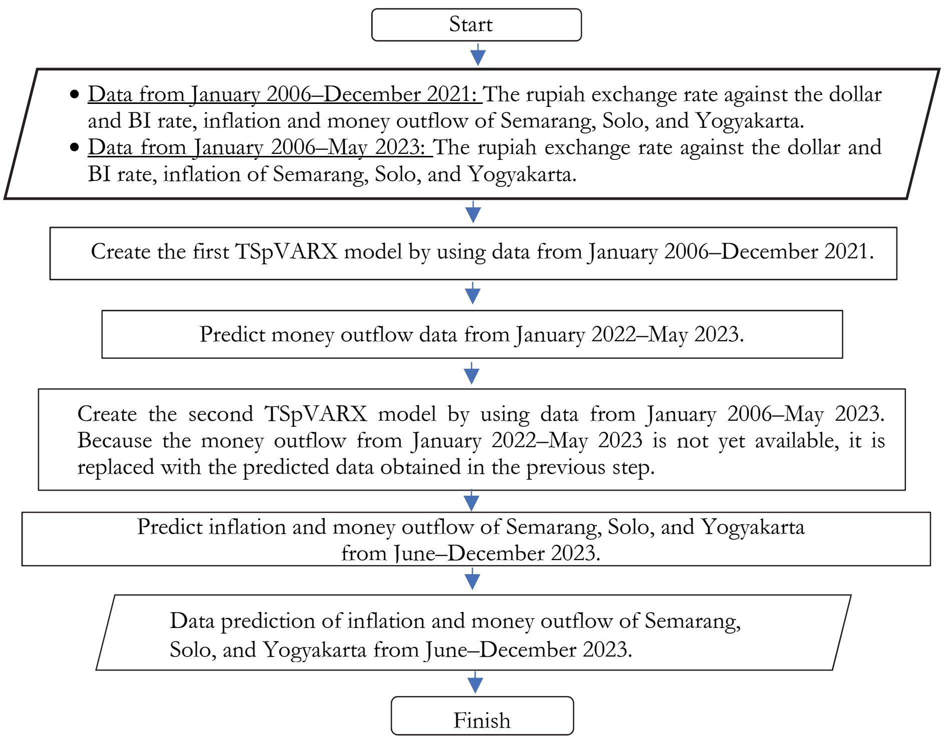
In our study, we create two TSpVARX models for predicting the inflation and money outflow of Yogyakarta, Solo, and Semarang. The first TSpVARX is for predicting money outflow data from January 2002 until February 2023. The Second TSpVARX is for predicting inflation and money outflow from June until December 2023. We are taking several stages to do it.

1. We create the first TSpVARX using data from January 2006–December 2021.
2. We predict money outflow data from January 2022–May 2023 using the first TSpVARX obtained from the first step. We do it because money outflow data is unavailable for January 2022–May 2023.
3. We create the second TSpVARX using data from January 2006–May 2023. We use the predicting results of the second step to complete money outflow data from January 2022–May 2023.
4. We predict the inflation and money outflow from June–December 2023 using the second TSpVARX from the third step.

Figure 3 shows us the flow chart of inflation and money outflow prediction in this study.

Figure 3

Flow chart of predicting application by using TSpVARX



The result of the simulation study

Evaluation of the sign precision of the TSpVARX model coefficient estimates

In this simulation, each repetition will produce 60 model coefficient estimators. Because there are 100 repetitions, there will be 6000 TSpVARX model coefficient estimators. Of the 6000 estimated model coefficients in each scenario, it will be calculated what percentage of the estimated coefficients already have the same sign as the true parameter. The greater the percentage produced, the better MLE is in producing coefficient estimators of TSpVARX in terms of sign accuracy. Table 1 shows the percentage of coefficient estimators of the TSpVARX that is equal to the true parameters according to the sample size and error covariance between endogenous variables. Table 1 shows that the overall percentage of sign similarity between the coefficient estimators and the true coefficient parameters is above 74% for all conditions of sample size and error covariance. If we look according to the sample size used, we can conclude that the percentage of sign similarity increases as the sample size increases.

Table 1

The percentage similarity of the sign of the estimated TSpVARX coefficients with the true parameters

Sample size	Error covariance	
	$Cov(e_ie_j) = 0.01$	$Cov(e_ie_j) = 0.09$
$n = 120$	75.315	74.900
$n = 240$	76.335	76.385
$n = 360$	77.915	77.465

Note: $Cov(e_ie_j)$ is error covariance between *i*-th and *j*-th endogenous variables.

Evaluation of the accuracy of the model coefficient estimation based on the results of the significance of the model parameter coefficients

The number of significant parameter coefficients in the model for all conditions is above 65% at the 5% significance level and above 69% at the 10% significance level. The number of significant model parameter coefficients when the error covariance between models is 0.01 is higher than when the error covariance between models is high. In addition, the larger the sample size used, the more significant the parameter coefficients. This shows that the MLE method is better used when the covariance error between equations is low and the sample size is large. However, the use of a sample size of 120 has also resulted in a percentage of significant parameter coefficients above 50%.

Table 2

The percentage of the number of significant parameter coefficients in the model

Significance level	Sample size	When $Cov(e_ie_j) = 0.01$	When $Cov(e_ie_j) = 0.09$
5%	120	65.52	65.25
	240	66.85	65.52
	360	68.10	68.12
10%	120	70.42	69.12
	240	71.07	69.83
	360	71.93	72.93

The evaluation of the accuracy of the estimated values of the TSpVARX parameter coefficients

The accuracy of the coefficient estimators in terms of the closeness of values will be evaluated by calculating the percentage of true parameters that fall into the 95% interval of the parameters coefficient model. We calculate the average estimator, lower bound, and upper bound of the interval resulting from 100 repetitions for each scenario. We calculate the percentage of true parameters that fall into the 95% interval for each scenario. The higher the percentage, the better the estimation results. Table 3 presents the percentage of true parameters that fall into the 95% interval for each scenario. In Table 3, we can see that the percentage of true parameters that have entered into the confidence interval formed is above 80% for each scenario performed. It indicates that empirically the MLE estimator is unbiased in estimating the TSpVARX model coefficients.

Table 3

Percentage of true parameters that fall within the 95% for each scenario

Sample size	Error covariance	
	$Cov(e_ie_j) = 0.01$	$Cov(e_ie_j) = 0.09$
n = 120	88.33	88.33
n = 240	81.67	83.33
n = 360	85.00	83.33

The result of TSpVARX application

The initial stage in time series modeling is to test the data stationary. If the data is not stationary at the level, then the data needs to be transformed into 1st or 2nd difference. We can see in Table 4 that the probability value (PV) produced in the stationary test on all variables is 0.000. At a significance level of 5%, we obtain that all PV is less than the significance level. Thus, we can conclude that all variables in this study are stationary.

Table 4

Stationary test output

Variable	PV	Conclusion
Inf_Se	0.0000	stationary
Inf_So	0.0000	stationary
Inf_Yo	0.0000	stationary
Out_Se	0.0000	stationary
Out_So	0.0000	stationary
Out_Yo	0.0000	stationary
Dep	0.0000	stationary
BI_rate	0.0000	stationary

Once we test data stationery, we identify the presence of spatial relationships between endogenous variables. We do it by looking at the significance of cross-correlation values. There is a spatial relationship if there is a significant correlation between one region's endogenous variable and another region's endogenous lag variable. The calculating result of the cross-correlation between endogenous variables in lag one can be seen in Table 5. Suppose the PV of the cross-correlation between variables is less than 5%. In that case, the cross-correlation between these variables is significant at the significance level of 5%. Table 5 shows that the PV of the cross-correlation between each region's inflation and other regions' inflation in lag one is less than 5%. It indicates that inflation in Semarang, Solo, and Yogyakarta is interrelated. Likewise, the PV of the cross-correlation between each region's money outflow and other regions' money outflow is less than 5%, so the money outflow of Semarang, Solo, and Yogyakarta is also interrelated. The spatial relationship also occurs between money outflow and inflation. It can be seen from the PV of the cross-correlation between Semarang's money outflow and Yogyakarta's inflation and the PV of the correlation between Solo's money outflow and Yogyakarta's inflation, which are less than 5%.

Table 5

Significance test of cross correlation based on endogenous variables

Endogenous variable	Inf_Se_{t-1}		Inf_So_{t-1}		Inf_Yo_{t-1}	
	cross-correlation	PV	cross-correlation	PV	cross-correlation	PV
Inf_Se	0.3035*	0.000	0.3664*	0.000	0.3652*	0.000
Inf_So	0.1939*	0.000	0.2900*	0.000	0.2610*	0.000
Inf_Yo	0.3211*	0.000	0.3518*	0.000	0.4189*	0.000
Out_Se	–0.1520*	0.044	–0.0941	0.2140	–0.2312*	0.002
Out_So	–0.1141	0.131	–0.0568	0.454	–0.1886*	0.012
Out_Yo	–0.1309	0.083	–0.0881	0.245	–0.2077*	0.006

(Table continues on the next page.)

(Continued.)

Endogenous variable	Out_Se_{t-1}		Out_So_{t-1}		Out_Yo_{t-1}	
	cross-correlation	PV	cross-correlation	PV	cross-correlation	PV
Inf_Se	–0.0524	0.490	0.0257	0.735	–0.0222	0.770
Inf_So	–0.0096	0.900	0.0038	0.960	0.0200	0.792
Inf_Yo	–0.0866	0.253	–0.0588	0.438	–0.0606	0.424
Out_Se	0.2846*	0.000	0.2735*	0.000	0.2946*	0.000*
Out_So	0.2678*	0.000	0.2884*	0.000	0.2782	0.000*
Out_Yo	0.2328*	0.002	0.2331*	0.002	0.2808	0.000*

* Significant with a significance level of 5%.

After we get the results of the inter-regional linkages in the previous section, the next step is calculating the spatial weighting used in the modeling. We use uniform spatial weighting because Semarang, Solo, and Semarang are easily interconnected. Here are the spatial weight values in this study:

$$\begin{aligned} & w_{\text{inf,inf}}(\text{Semarang,Solo}) = w_{\text{inf,inf}}(\text{Semarang, Yogyakarta}) = w_{\text{inf,inf}}(\text{Solo, Yogyakarta}) \\ & = w_{\text{inf,Out}}(\text{Semarang,Solo}) = w_{\text{inf,Out}}(\text{Semarang, Yogyakarta}) = w_{\text{inf,Out}}(\text{Solo, Yogyakarta}) \\ & = w_{\text{Out, Inf}}(\text{Semarang,Solo}) = w_{\text{Out,Inf}}(\text{Semarang, Yogyakarta}) = w_{\text{Out,Inf}}(\text{Solo, Yogyakarta}) \\ & = w_{\text{Out, Out}}(\text{Semarang,Solo}) = w_{\text{Out, Out}}(\text{Semarang, Yogyakarta}) = w_{\text{Out,Out}}(\text{Solo, Yogyakarta}) = \frac{1}{2}. \end{aligned}$$

We give an example of $w_{\text{inf,Out}}(\text{Semarang, Yogyakarta}) = \frac{1}{2}$ for interpreting the symbol of the spatial weight that we write above. It means the spatial weight for the relationship between Semarang's inflation and Yogyakarta's money outflow is $\frac{1}{2}$.

Once we have determined the spatial weight, the next step is determining the autoregressive order (*p*). We determine it by selecting the *p* order that produces the smallest AIC on the SpVAR(1,*p*) model. We present the AIC of the SpVAR(1,*p*) model with and without constant in Table 6. Table 6 shows that the smallest AIC is obtained from the SpVAR(1,*p*) with *p*=1. It means that SpVARX and TSpVARX modeling will use a *p*-order of 1.

Table 6

AIC model SpVAR(1,*p*) according to order *p*

SpVAR(1,*p*)	*p*	AIC
With constant	1	5353.772
	2	5360.290
	3	5421.603
	4	5493.338
Without constant	1	5423.061
	2	5418.331
	3	5459.231
	4	5522.384

The next stage is to find the *q* order used in SpVARX and TSpVARX modeling. We choose *q* that results the smallest AIC of SpVARX. The results can be seen in Table 7. We get the smallest AIC when *q*=4. Therefore, the *q* order to be used is 4.

Table 7

The AIC of SpVARX(1,1,*q*) according to order *q*

SpVARX (1,1,*q*) with constant	AIC
SpVARX (1,1,1)	5338.874
SpVARX (1,1,2)	5319.166
SpVARX (1,1,3)	5300.985
SpVARX (1,1,4)	5290.619

After we get orders *p*=1 and *q*=4, the next step is to do SpVARX(1,1,4) modeling. Here is the model obtained:

$$\begin{aligned}\widehat{Inf}\_Se_t =\ & 0.017-9.170e^{-5}Dep_{t-1}+5.304e^{-1}BI\_rate_{t-1}+4.216e^{-5}Dep_{t-2}-0.6225BI\_rate_{t-2}-1{,}268e^{-4}Dep_{t-3}\\ & -1.821e^{-1}BI\_rate_{t-3}-2.653e^{-5}Dep_{t-4}+3.08e^{-5}BI\_rate_{t-4}+0.0559Inf\_Se_{t-1}\\ & +0.271\left(0.5Inf\_So_{t-1}+0.5Inf\_Yo_{t-1}\right)-3.511e^{-5}Out\_Se_{t-1}+5.27e^{-5}\left(0.5Out\_So_{t-1}+0.5Out\_Yo_{t-1}\right)\end{aligned} \tag{9a}$$

$$\begin{aligned}\widehat{Inf}\_So_t =\ & 0.047-1.132e^{-4}Dep_{t-1}+4.530e^{-1}BI\_rate_{t-1}+1.6331e^{-5}Dep_{t-2}-0.545BI\_rate_{t-2}-5.875e^{-5}Dep_{t-3}\\ & -1.8326e^{-1}BI\_rate_{t-3}+1.109e^{-5}Dep_{t-4}+3.053e^{-1}BI\_rate_{t-4}+0.226Inf\_So_{t-1}\\ & +9.387e^{-3}\left(0.5Inf\_Se_{t-1}+0.5Inf\_Yo_{t-1}\right)-1.160e^{-1}Out\_So_{t-1}+7.616e^{-5}\left(0.5Out\_Se_{t-1}+0.5Out\_Yo_{t-1}\right)\end{aligned} \tag{9b}$$

$$\begin{aligned}\widehat{Inf}\_Yo_t =\ & -0.122-7.375e^{-5}Dep_{t-1}+0.4702BI\_rate_{t-1}-3.122e^{-5}Dep_{t-2}-0.4034BI\_rate_{t-2}-9.144Dep_{t-3}\\ & -0.2093BI\_rate_{t-3}-1.067e^{-4}Dep_{t-4}+0.209BI\_rate_{t-4}+0.172Inf\_Yo_{t-1}\\ & +0.1083\left(0.5Inf\_Se_{t-1}+0.5Inf\_So_{t-1}\right)-3.117e^{-5}Out\_Yo_{t-1}+2.250e^{-5}\left(0.5Out\_Se_{t-1}+0.5Out\_So_{t-1}\right)\end{aligned} \tag{9c}$$

$$\begin{aligned}\widehat{Out}\_Se_t =\ & 3250.683+0.649Dep_{t-1}+265.721e^{-1}BI\_rate_{t-1}-0.100Dep_{t-2}-1004.684BI\_rate_{t-2}+0.196Dep_{t-3}\\ & +655.062BI\_rate_{t-3}-0.336Dep_{t-4}-203.249BI\_rate_{t-4}-204.070Inf\_Se_{t-1}\\ & -110.409\left(0.5Inf\_So_{t-1}+0.5Inf\_Yo_{t-1}\right)+0.087Out\_Se_{t-1}+0.121\left(0.5Out\_So_{t-1}+0.5Out\_Yo_{t-1}\right)\end{aligned} \tag{9d}$$

$$\begin{aligned}\widehat{Out}\_So_t =\ & 1406.509+0.319Dep_{t-1}+207.229BI\_rate_{t-1}-0.042Dep_{t-2}-515.656BI\_rate_{t-2}+0.114Dep_{t-3}\\ & +288.940BI\_rate_{t-3}-0.193Dep_{t-4}-94.002BI\_rate_{t-4}+0.220Inf\_So_{t-1}\\ & -156.216\left(0.5Inf\_Se_{t-1}+0.5Inf\_Yo_{t-1}\right)+0.561Out\_So_{t-1}-0.254\left(0.5Out\_Se_{t-1}+0.5Out\_Yo_{t-1}\right)\end{aligned} \tag{9e}$$

$$\begin{aligned}\widehat{Out}\_Yo_t =\ & 1797.218+0.245Dep_{t-1}+176.237BI\_rate_{t-1}-0.074Dep_{t-2}-292.911BI\_rate_{t-2}+0.115Dep_{t-3}\\ & +161.984BI\_rate_{t-3}-0.241Dep_{t-4}-178.757BI\_rate_{t-4}-105.952Inf\_Yo_{t-1}\\ & -89.840\left(0.5Inf\_Se_{t-1}+0.5Inf\_So_{t-1}\right)+0.278Out\_Yo_{t-1}-0.114\left(0.5Out\_Se_{t-1}+0.5Out\_So_{t-1}\right)\end{aligned} \tag{9f}$$

To proceed with TSpVARX modeling, we first need to test whether there is a nonlinear relationship between endogenous variables. We used Ramsey's reset test to test whether inflation and money outflow variables in Semarang, Solo, and Yogyakarta can be modelled nonlinearly. The null hypothesis of Ramsey's reset test is that the relationship between endogenous variables and predetermined is linear, while the alternative hypothesis is that the relationship between endogenous variables and predetermined variables is not linear. Suppose the Ramsey reset test produces a PV less than the significance level. In that case, we reject the null hypothesis, meaning that the relationship between endogenous and predetermined variables is not linear. The results of nonlinearity testing using Ramsey's reset test can be seen in Table 8. We use a significance level of 5%. We can see that $\hat{Inf}\_Se_t$, $\hat{Inf}\_So_t$, $\hat{Out}\_Se_t$, $\hat{Out}\_So_t$, and $\hat{Out}\_Yo_t$ gets a PV of less than 5% on Ramsey's reset test. Thus, we can conclude that the relationship between those five endogenous variables and the predetermined variable is not linear. Since most endogenous variables have a nonlinear relationship with predetermined variables, we will proceed to TSpVARX modeling.

Table 8

Linearity testing results using Ramsey's reset test

Endogenous variable	PV	Decision
Inf_Se	0.01866	nonlinear
Inf_So	0.01985	nonlinear
Inf_Yo	0.1803	linear
Out_Se	0.004965	nonlinear
Out_So	0.01035	nonlinear
Out_Yo	0.007942	nonlinear

We will use the orders *p*=1 and *q*=4 as previously obtained to create TSpVARX(1,1,4). Because the order *p*=1, the maximum delay value (*d*) is one. We first set an endogenous lag variable as a threshold variable. We select it based on the smallest AIC value TSpVARX generates for each possible threshold variable. The AIC of the TSpVARX model according to order *d* can be seen in Table 9. Based on Table 9, TSpVARX (1,1,4) produces the smallest AIC when used Out_So_{t-1} as the threshold variable.

Table 9

AIC of TSpVARX (1,1,4) with constant according to a variable threshold

Threshold variable	AIC			
	TSpVARX (1,1,1)	TSpVARX (1,1,2)	TSpVARX (1,1,3)	TSpVARX (1,1,4)
	with constant			
Inf_Se_{t-1}	5327.245	5322.36	5301.526	5199.983
Inf_So_{t-1}	5294.394	5337.618	5301.165	5080.299
Inf_Yo_{t-1}	5322.574	5323.217	5318.226	5210.329
Out_Se_{t-1}	5197.598	5193.354	5175.769	5174.832
Out_So_{t-1}	5236.664	5204.610	5151.338	5046.560
Out_Yo_{t-1}	5208.774	5192.622	5154.233	5083.241

The next step is to model the inflation and money outflow of Semarang, Solo, and Yogyakarta using TSpVARX(1,1,4) Out_So_{t-1} as the threshold variable. We get the estimated threshold value $(\hat{\varsigma})$ of Out_So_{t-1} is 12 billion rupiahs. The TSpVARX (1,1,4) model formed is as follows:

when $Out\_So_{t-1} \leq 12\,\text{billion rupiahs}$,

$$\begin{aligned}\widehat{Inf}\_Se_t =& -1.429 - 1.02e^{-3} Dep_{t-1} + 1.842 BI\_rate_{t-1} + 7.189e^{-4} Dep_{t-2} - 4.794 BI\_rate_{t-2} - 3.202e^{-4} Dep_{t-3} \\ & + 4.545 BI\_rate_{t-3} + 1.561e^{-4} Dep_{t-4} - 1.355 BI\_rate_{t-4} + 1.127e^{-3}\, Out\_Se_{t-1} \\ & - 4.606e^{-4}\left(0.5 Out\_So_{t-1} + 0.5 Out\_Yo_{t-1}\right) - 9.27e^{-2} Inf\_Se_{t-1} - 6.272e^{-3}\left(0.5 Inf\_So_{t-1} + 0.5 Inf\_Yo_{t-1}\right)\end{aligned} \tag{10a}$$

$$\begin{aligned}\widehat{Inf}\_So_t =& -0.653 - 6.133e^{-4} Dep_{t-1} + 1.283 BI\_rate_{t-1} + 2.717e^{-5} Dep_{t-2} - 3.792 BI\_rate_{t-2} - 5.557e^{-5} Dep_{t-3} \\ & + 4.358 BI\_rate_{t-3} + 1.170e^{-4} Dep_{t-4} - 1.724 BI\_rate_{t-4} - 6.894e^{-3}\, Out\_So_{t-1} \\ & + 6.710e^{-4}\left(0.5 Out\_Se_{t-1} + 0.5 Out\_Yo_{t-1}\right) - 0.307 Inf\_So_{t-1} + 0.122\left(0.5 Inf\_Se_{t-1} + 0.5 Inf\_Yo_{t-1}\right)\end{aligned} \tag{10b}$$

$$\begin{aligned}\widehat{Inf}\_Yo_t =& -0.396 - 4.032e^{-4} Dep_{t-1} - 2.375 BI\_rate_{t-1} + 3.118e^{-4} Dep_{t-2} + 3.46 BI\_rate_{t-2} - 3.76e^{-4} Dep_{t-3} \\ & - 0.648 BI\_rate_{t-3} - 2.522e^{-4} Dep_{t-4} - 0.382 BI\_rate_{t-4} + 8.845e^{-4}\, Out\_Yo_{t-1} \\ & - 9.447e^{-4}\left(0.5 Out\_Se_{t-1} + 0.5 Out\_So_{t-1}\right) + 0.843 Inf\_Yo_{t-1} - 0.285\left(0.5 Inf\_Se_{t-1} + 0.5 Inf\_So_{t-1}\right)\end{aligned} \tag{10c}$$

$$\begin{aligned}Out\_Se =& 41.543 + 0.504 Dep_{t-1} + 2556.422 BI\_rate_{t-1} - 0.653 Dep_{t-2} - 2903.201 BI\_rate_{t-2} + 0.615 Dep_{t-3} \\ & - 943.984 BI\_rate_{t-3} + 0.088 Dep_{t-4} + 1293.163 BI\_rate_{t-4} + 2.882e^{-4}\, Out\_Se_{t-1} \\ & - 2.076\left(0.5 Out\_So_{t-1} + 0.5 Out\_Yo_{t-1}\right) - 58.074 Inf\_Se_{t-1} - 183.043\left(0.5 Inf\_So_{t-1} + 0.5 Inf\_Yo_{t-1}\right)\end{aligned} \tag{10d}$$

$$\begin{aligned}Out\_So =& 262.782 + 0.186 Dep_{t-1} + 221.518 BI\_rate_{t-1} - 0.0397 Dep_{t-2} - 238.301 BI\_rate_{t-2} + 0.120 Dep_{t-3} \\ & - 361.201 BI\_rate_{t-3} + 0.0373 Dep_{t-4} + 336.560 BI\_rate_{t-4} + 5.996e^{-4}\, Out\_So_{t-1} \\ & + 0.417\left(0.5 Out\_Se_{t-1} + 0.5 Out\_Yo_{t-1}\right) + 33.008 Inf\_So_{t-1} - 25.883\left(0.5 Inf\_Se_{t-1} + 0.5 Inf\_Yo_{t-1}\right)\end{aligned} \tag{10e}$$

$$\begin{aligned}Out\_Yo =& 755.110 + 0.489 Dep_{t-1} + 2323.844 BI\_rate_{t-1} - 0.504 Dep_{t-2} - 3052.534 BI\_rate_{t-2} + 0.568 Dep_{t-3} \\ & - 475.913 BI\_rate_{t-3} + 0.179 Dep_{t-4} + 1099.791 BI\_rate_{t-4} - 0.606\, Out\_Yo_{t-1} \\ & + 4.590\left(0.5 Out\_Se_{t-1} + 0.5 Out\_So_{t-1}\right) - 58.767 Inf\_Yo_{t-1} + 29.122\left(0.5 Inf\_Se_{t-1} + 0.5 Inf\_So_{t-1}\right)\end{aligned} \tag{10f}$$

when $Out\_So_{t-1} > 12\,\text{billion rupiahs}$,

$$\begin{aligned}\widehat{Inf}\_Se_t =& 0.113 - 3.016e^{-5} Dep_{t-1} + 0.562 BI\_rate_{t-1} + 1.298e^{-5} Dep_{t-2} - 0.518 BI\_rate_{t-2} - 9.985e^{-5} Dep_{t-3} \\ & - 0.286 BI\_rate_{t-3} - 1.606e^{-4} Dep_{t-4} + 0.275 BI\_rate_{t-4} - 6.133e^{-5}\, Out\_Se_{t-1} \\ & + 8.792e^{-5}\left(0.5 Out\_So_{t-1} + 0.5 Out\_Yo_{t-1}\right) + 0.0796 Inf\_Se_{t-1} + 0.218\left(0.5 Inf\_So_{t-1} + 0.5 Inf\_Yo_{t-1}\right)\end{aligned} \tag{10g}$$

$$\begin{aligned}\widehat{Inf}\_So_t =& 0.0430 - 6.615e^{-5} Dep_{t-1} + 0.539 BI\_rate_{t-1} + 6.874e^{-5} Dep_{t-2} - 0.501 BI\_rate_{t-2} - 1.534e^{-4} Dep_{t-3} \\ & - 0.462 BI\_rate_{t-3} + 3.345e^{-4} Dep_{t-4} + 0.461 BI\_rate_{t-4} - 1.385e^{-4}\, Out\_So_{t-1} \\ & + 6.7550e^{-5}\left(0.5 Out\_Se_{t-1} + 0.5 Out\_Yo_{t-1}\right) + 0.205 Inf\_So_{t-1} + 0.057\left(0.5 Inf\_Se_{t-1} + 0.5 Inf\_Yo_{t-1}\right)\end{aligned} \tag{10h}$$

$$\begin{aligned}\widehat{Inf}\_Yo_t =& -0.114 - 1.521e^{-5} Dep_{t-1} + 0.538 BI\_rate_{t-1} - 4.056e^{-5} Dep_{t-2} - 0.383 BI\_rate_{t-2} - 1.155e^{-4} Dep_{t-3} \\ & - 0.356 BI\_rate_{t-3} - 1.064e^{-4} Dep_{t-4} + 0.275 BI\_rate_{t-4} - 6.49e^{-5}\, Out\_Yo_{t-1} \\ & + 2.371e^{-5}\left(0.5 Out\_Se_{t-1} + 0.5 Out\_So_{t-1}\right) + 0.184 Inf\_Yo_{t-1} + 0.1\left(0.5 Inf\_Se_{t-1} + 0.5 Inf\_So_{t-1}\right)\end{aligned} \tag{10i}$$

$$\begin{aligned} Out\_Se &= 3417.830 + 0.735 Dep_{t-1} + 419.148 BI\_rate_{t-1} - 0.204 Dep_{t-2} - 1075.496 BI\_rate_{t-2} + 0.225 Dep_{t-3} \\ &+ 550.124 BI\_rate_{t-3} - 0.398 Dep_{t-4} - 176.963 BI\_rate_{t-4} + 0.0429 Out\_Se_{t-1} \\ &+ 0.097\left(0.5 Out\_So_{t-1} + 0.5 Out\_Yo_{t-1}\right) - 241.192 Inf\_Se_{t-1} - 117.030\left(0.5 Inf\_So_{t-1} + 0.5 Inf\_Yo_{t-1}\right) \end{aligned} \quad (10j)$$

$$\begin{aligned} Out\_So &= 1463.658 + 0.428 Dep_{t-1} + 290.523 BI\_rate_{t-1} - 0.114 Dep_{t-2} - 528.851 BI\_rate_{t-2} + 0.181 Dep_{t-3} \\ &+ 211.221 BI\_rate_{t-3} - 0.243 Dep_{t-4} - 77.526 BI\_rate_{t-4} + 0.564 Out\_So_{t-1} \\ &- 0.298\left(0.5 Out\_Se_{t-1} + 0.5 Out\_Yo_{t-1}\right) + 14.728 Inf\_So_{t-1} - 201.292\left(0.5 Inf\_Se_{t-1} + 0.5 Inf\_Yo_{t-1}\right) \end{aligned} \quad (10k)$$

$$\begin{aligned} Out\_Yo &= 1880.905 + 0.267 Dep_{t-1} + 279.944 BI\_rate_{t-1} - 0.169 Dep_{t-2} - 284.393 BI\_rate_{t-2} + 0.169 Dep_{t-3} \\ &+ 85.722 BI\_rate_{t-3} - 0.353 Dep_{t-4} - 204.742 BI\_rate_{t-4} + 0.239 Out\_Yo_{t-1} \\ &- 0.138\left(0.5 Out\_Se_{t-1} + 0.5 Out\_So_{t-1}\right) - 183.772 Inf\_Yo_{t-1} - 56.003\left(0.5 Inf\_Se_{t-1} + 0.5 Inf\_So_{t-1}\right) \end{aligned} \quad (10l)$$

We will compare the predicting performance between SpVARX(1,1,4) and TSpVARX(1,1,4). We perform predicting for the data testing period using equations (9a) to (9f) for predicting based on SpVARX (1,1,4) and equations (10a) to (10l) for predicting based on TSpVARX (1,1,4). There are 12 months of testing data (January–December 2021), which will be used to evaluate the predicting performance of the TSPVARX model compared to SpVARX in forecasting inflation and money outflow for Semarang, Solo, and Yogyakarta.

After that, we calculate the RMSE and SMAPE of SpVARX and TSpVARX based on the prediction results for the testing data period. RMSE and SMAPE calculation results can be seen in Table 10. Four of the six endogenous variables get the smallest RMSE from TSpVARX, which are Inf_Se, Inf_So, Inf_Yo, and Out_Se. Three of the six endogenous variables get the smallest SMAPE value from TSpVARX, which are Inf_Se, Inf_So, and Inf_Yo. It indicates that most inflation predictions are better with TSpVARX than SpVARX.

Table 10

Predicting performance of SpVARX and TSpVARX according to the endogenous variable and RMSE

Endogenous variable	SpVAR (1,1,4)	SpVARX (1,1,4)	TSpVARX (1,1,4)
		RMSE	
Inf_Se	0.109	0.177	**0.062**
Inf_So	0.07	0.098	**0.069**
Inf_Yo	0.103	0.1	**0.065**
Out_Se	2662536.024	1963363.586	**1954042.647**
Out_So	598927.125	351958.168	394884.645
Out_Yo	650487.322	624390.481	736891.300
		SMAPE	
Inf_Se	116.315	122.137	**67.218**
Inf_So	92.893	98.672	**66.243**
Inf_Yo	111.275	111.412	**61.657**
Out_Se	53.545	51.083	61.981
Out_So	60.062	44.485	62.619
Out_Yo	68.202	66.245	82.308

In addition to calculating the RMSE and SMAPE for each forecasting variable, we also display the Akaike information criterion (AIC), the average of RMSE, and the average of SMAPE. This aims to make it easier to see what is the best model in general for predicting inflation and money outflow in Semarang, Solo, and Yogyakarta. Because there is a considerable difference between the RMSE values of inflation and money outflow, we find the average of the logarithm of RMSE. Table 11 shows the comparison of AIC, average log RMSE, and average SMAPE between TSpVARX and SpVARX models. Based on Table 11, we can see that TSpVARX gets the smallest value for AIC, average log RMSE, and average SMAPE. It indicates that the TSpVARX model is the best compared to SpVAR and SpVARX to predict inflation and money outflow in Semarang, Solo, and Yogyakarta.

Table 11

The comparison of AIC, average log RMSE, and average SMAPE between SpVARX and TSpVARX models

Model	AIC	Average log RMSE	Average SMAPE
SpVAR(1,1,4)	5353.772	5.737	83.715
SpVARX(1,1,4)	5290.619	5.709	82.339
TSpVARX(1,1,4)	5046.56	5.447	67.004

In the introduction, it has been explained that inflation and money outflow have a reciprocal relationship, get influenced by metric exogenous variables such as interest rates and exchange rates, have inter-regional linkages, and have a nonlinear relationship with predetermined variables. This is then confirmed by modeling performance measurement which concludes that TSpVARX modeling performance is better than SpVAR and SpVARX. Therefore, it makes us understand that the phenomena of inflation and money outflow are suitable to be modeled with TSpVARX.

Next, we will predict inflation and money outflow of Yogyakarta, Solo, and Semarang from June until December 2023 using TSpVARX. We get the forecast result in Table 12. Based on Table 12, we get some information. From June until December 2023, the highest inflation prediction in Solo and Yogyakarta will be in December. It can be triggered because of the price increase that usually occurs a few days before Christmas and New Year's holidays (Nairobi–Caroline 2021, Farandy 2020). In line with this, money outflow is also predicted to be the highest in December. It can occur due to increased community needs before Christmas and New Year, triggering an increase in bank money withdrawals so that the money outflow also increases (Monica et al. 2021, Prastyo et al. 2018).

Table 12

Prediction of Semarang's inflation (Inf_Se), Solo's inflation (Inf_So), Yogyakarta's inflation (Inf_Yo), Semarang's money outflow (Out_Se), Solo's money outflow (Out_So), and Yogyakarta's money outflow (Out_Yo) from March until December 2023

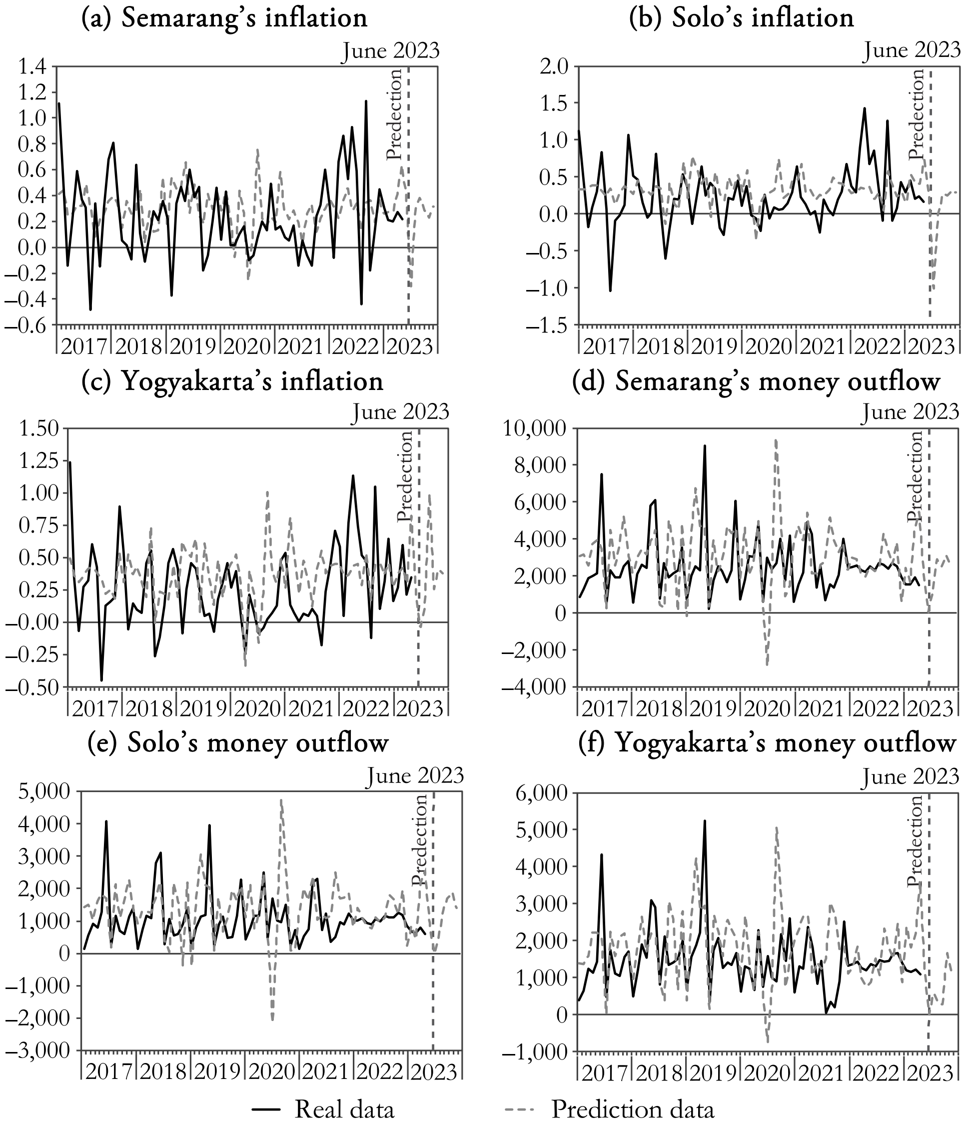
Month	Inf_Se	Inf_So	Inf_Yo	Out_Se	Out_So	Out_Yo
June	0.384	0.300	0.236	2181.37	955.91	1196.104
July	–0.301	–0.920	–0.031	17.00	2.00	7
September	0.301	0.056	0.119	1082.93	458.09	585.876
October	0.382	0.329	0.998	2897.73	1339.36	363.349
November	0.286	0.306	0.262	2495.48	1694.17	282.177
December	0.232	0.376	0.413	3088.31	1896.98	1657.115

Based on the prediction results, the government needs to take policies to avoid high inflation rates in December, when there are Christmas celebrations and the time before the turn of the year. Some things that can be done include increasing Bank Indonesia's benchmark interest rate, which triggers an increase in savings and deposit rates. It attracts people to invest their funds in banks (Rahman et al. 2022, Katmas–Indarningsih 2022, Amanda et al. 2023). Thus, the desire to do excessive consumptive things that trigger inflation before Christmas and New Year can be reduced. The next policy is to ensure that the needs related to Christmas and New Year celebrations are available and easily accessible to the public. It is useful to avoid scarcity of goods that can trigger inflation (Ubide 2022, Priwiningsih–Abidin 2022, Nugraha et al. 2023). Supervision of the distribution of goods is tightened, starting from the production center area to the destination of the area where the goods will be sold. It can avoid the existence of the mafia that can hoard goods so that the price of goods becomes expensive (Huda–Sidiq 2023, Camila et al. 2022).

Visually, a comparison between real and prediction data results using TSpVARX can be seen in Figure 4. The black line depicts the movement of real data, while the dotted grey line depicts the movement of prediction data. From Figure 4, we can see that some data predictions are far from the original data. It indicates that other variables must still be involved in the modelling to make the prediction results closer to real data. Other variables that can affect inflation and money outflow movements in Indonesia include rising fuel and electricity prices and seasonal influences such as the new school year, Ramadan, Eid al-Fitr, etc. (Sohibien 2018, Anugrah–Pratama 2018, Sumarminingsih et al. 2021). However, some fluctuation patterns of prediction data are already the same as the real data.

Figure 4

Comparison between real and prediction data



Conclusion

In this research, we proposed the TSpVARX to predict inflation and money outflow in Semarang, Solo, and Yogyakarta. We found that spatial aspects must be included in inflation and money outflow modeling in Semarang, Solo, and Yogyakarta. It is shown by the significant cross-correlation between inflation and lag inflation; money outflow and lag money outflow; and money outflow and lag inflation.

Based on the smallest AIC of SpVAR and SpVARX, the maximum temporal lag order is one, and the maximum lag of the exogenous variable is four. It indicates that the rise and fall of Bank Indonesia's benchmark interest rate and exchange rate depreciation have a more prolonged impact on inflation and money outflow than fluctuations in inflation and money outflow in the previous period. From the results of Ramsey's reset test, we find that aspects of nonlinearity need to be considered in inflation and money outflow modeling. It is critical to be able to continue modeling using TSpVARX.

We find that TSpVARX performs better than SpVARX in predicting inflation in Semarang, Solo, Yogyakarta, and money outflow in Semarang. In the predicting section, we get insight that during the June–December 2023 period, the highest inflation in Solo and Yogyakarta is predicted to occur in December 2023. Meanwhile, the highest money outflow in Semarang, Solo, and Yogyakarta are also predicted to occur in December 2023.

This research still has limitations that can be an idea for future research. Although empirically, in this study, TSpVARX is better than SpVARX in forecasting inflation and money outflow, future research can be carried out in simulation studies to strengthen the results that when there is a nonlinear relationship between variables, TSpVARX is better than SpVARX. Accommodating nonmetric variables such as the global crisis of 2008, Covid-19 pandemic, seasonal patterns, etc. in modeling inflation and money supply is interesting. In addition, if possible, future research should increase the amount of data so that TSpVARX models can be formed in more than two regimes.

Appendix

The proof of theorem 1

Based on equation (7), $\hat{\mathbf{b}}^{(g)}_{MLE,TSpVARX}$ can be found with the following formula:

$$\hat{\mathbf{b}}^{(g)}_{MLE,TSpVARX}=\left(\left(\text{Vec}\left(\mathbf{Fb}\right)^{(g)}\right)'\left(\hat{\Sigma}^{(g)}_{OLS,SpVARX}\otimes\mathbf{I}\right)\text{Vec}\left(\mathbf{Fb}\right)^{(g)}\right)^{-1}\left(\text{Vec}\left(\mathbf{Fb}\right)^{(g)}\right)'\left(\hat{\Sigma}^{(g)}_{OLS,SpVARX}\otimes\mathbf{I}\right)^{-1}\text{Vec}\left(\mathbf{Y}\right)^{(g)} \quad (11)$$

Equation (11) can be further decomposed into the following equation:

$$\begin{aligned}\hat{\mathbf{b}}^{(g)}_{MLE,TSpVARX}=&\left(\left(\text{Vec}\left(\mathbf{Fb}\right)^{(g)}\right)'\left(\hat{\Sigma}^{(g)}_{OLS,SpVARX}\otimes\mathbf{I}\right)^{-1}\text{Vec}\left(\mathbf{Fb}\right)^{(g)}\right)^{-1}\left(\text{Vec}\left(\mathbf{Fb}\right)^{(g)}\right)'\\&\times\left(\hat{\Sigma}^{(g)}_{OLS,SpVARX}\otimes\mathbf{I}\right)^{-1}\left(\text{Vec}\left(\mathbf{Fb}\right)^{(g)}\hat{\mathbf{b}}^{(g)}_{MLE,TSpVARX}+\mathbf{u}^{(g)}\right),\end{aligned} \quad (12)$$

where:

$\hat{\mathbf{b}}^{(g)}_{MLE,TSpVARX}$ is the vector of the coefficient estimator of the model in the *g*-th regime,

$\mathbf{u}^{(g)}$ is the error of the model in the *g*-th regime when we use the true parameter $\mathbf{b}^{(g)}_{TSpVARX}$.

Pawitan (2001) stated that the coefficient estimators obtained from MLE will be consistent if the estimators converge in probability to their parameters. Convergence testing of $\hat{\mathbf{b}}^{(g)}_{MLE,TSpVARX}$ can be done by showing that $\lim_{NKT^{(g)}\to\infty}\text{P}\left(\left|\hat{\mathbf{b}}^{(g)}_{MLE,TSpVARX}-\mathbf{b}^{(g)}_{TSpVARX}\right|>\in\right)=0$ or $\text{plim}\,\hat{\mathbf{b}}^{(g)}_{MLE,TSpVARX}=\mathbf{b}^{(g)}_{TSpVARX}$ (Lütkepohl 2005). Equation (12) can be decomposed into:

$$\begin{aligned}\hat{\mathbf{b}}^{(g)}_{MLE,TSpVARX}=&\,\mathbf{b}^{(g)}_{TSpVARX}+\left(\left(\text{Vec}\left(\mathbf{Fb}\right)^{(g)}\right)'\left(\hat{\Sigma}^{(g)}_{OLS,SpVARX}\otimes\mathbf{I}\right)^{-1}\text{Vec}\left(\mathbf{Fb}\right)^{(g)}\right)^{-1}\\&\times\left(\text{Vec}\left(\mathbf{Fb}\right)^{(g)}\right)'\left(\hat{\Sigma}^{(g)}_{OLS,SpVARX}\otimes\mathbf{I}\right)^{-1}\mathbf{u}^{(g)}.\end{aligned} \quad (13)$$

Based on equation (13), the consistency proof of $\hat{\mathbf{b}}^{(g)}_{MLE,TSpVARX}$ is as follows:

$$\begin{aligned}\text{plim}\,\hat{\mathbf{b}}^{(g)}_{MLE,TSpVARX}=&\,\mathbf{b}^{(g)}_{TSpVARX}+\text{plim}\left(\left(\frac{\left(\text{Vec}\left(\mathbf{Fb}\right)^{(g)}\right)'\left(\hat{\Sigma}^{(g)}_{OLS,SpVARX}\otimes\mathbf{I}\right)^{-1}\text{Vec}\left(\mathbf{Fb}\right)^{(g)}}{NKT^{(g)}}\right)^{-1}\right)\\&\times\text{plim}\left(\frac{\left(\text{Vec}\left(\mathbf{Fb}\right)^{(g)}\right)'\left(\hat{\Sigma}^{(g)}_{OLS,SpVARX}\otimes\mathbf{I}\right)^{-1}\mathbf{u}^{(g)}}{NKT^{(g)}}\right).\end{aligned} \quad (14)$$

Due to the fixed assumption of $\left(\text{Vec}\left(\mathbf{Fb}\right)^{(g)}\right)'\left(\hat{\boldsymbol{\Sigma}}^{(g)}_{OLS,SpVARX}\otimes\mathbf{I}\right)^{-1}\text{Vec}\left(\mathbf{Fb}\right)^{(g)}$ and the random assumption of $\mathbf{u}^{(g)}$, the equation (14) can be decomposed into:

$$\text{plim}\,\hat{\mathbf{b}}^{(g)}_{MLE,TSpVARX}=\mathbf{b}^{(g)}_{TSpVARX}+\text{plim}\left(\left(\frac{\left(\text{Vec}\left(\mathbf{Fb}\right)^{(g)}\right)'\left(\hat{\boldsymbol{\Sigma}}^{(g)}_{OLS,SpVARX}\otimes\mathbf{I}\right)^{-1}\text{Vec}\left(\mathbf{Fb}\right)^{(g)}}{NKT^{(g)}}\right)^{-1}\right).\mathbf{0}$$

$$\text{plim}\,\hat{\mathbf{b}}^{(g)}_{MLE,TSpVARX}=\mathbf{b}^{(g)}_{TSpVARX} \tag{15}$$

Thus theorem 1 is proved. □

The proof of theorem 2

Estimation of the TSpVARX model coefficients in the *g*-th regime with MLE is sought by maximizing the log-likelihood function in the *g*-th regime. We can write as follows:

$$S\left(\hat{\mathbf{b}}^{(g)}_{MLE,TSpVARX}\right)=\frac{dl\left(\mathbf{b}^{(g)}_{TSpVARX}\,|\,\boldsymbol{\Omega}^{(g)}\right)}{d\left(\mathbf{b}^{(g)}_{TSpVARX}\right)'}=\mathbf{0}\,. \tag{16}$$

$S\left(\hat{\mathbf{b}}^{(g)}_{MLE,TSpVARX}\right)$ can be approximated by the following Taylor series:

$$S\left(\hat{\mathbf{b}}^{(g)}_{MLE,TSpVARX}\right)=S\left(\mathbf{b}^{(g)}_{0,TSpVARX}\right)+\frac{dS\left(\mathbf{b}^{(g)}_{0,TSpVARX}\right)}{d\mathbf{b}^{(g)}_{TSpVARX}}\left(\hat{\mathbf{b}}^{(g)}_{MLE,TSpVARX}-\mathbf{b}^{(g)}_{0,TSpVARX}\right)=\mathbf{0}\,, \tag{17}$$

where:

$\mathbf{b}^{(g)}_{0,TSpVARX}$ is parameter value of $\mathbf{b}^{(g)}_{TSpVARX}$,

$\hat{\mathbf{b}}^{(g)}_{MLE,TSpVARX}$ is parameter estimation of $\mathbf{b}^{(g)}_{TSpVARX}$.

Equation (17) can be translated into the following:

$$\left(\hat{\mathbf{b}}^{(g)}_{MLE,TSpVARX}-\mathbf{b}^{(g)}_{0,TSpVARX}\right)=-H\left(\mathbf{b}^{(g)}_{0,TSpVARX}\right)^{-1}S\left(\mathbf{b}^{(g)}_{0,TSpVARX}\right)\,, \tag{18}$$

where:

$$H\left(\mathbf{b}^{(g)}_{0,TSpVARX}\right)=\frac{dS\left(\mathbf{b}^{(g)}_{0,TSpVARX}\right)}{d\mathbf{b}^{(g)}_{TSpVARX}}$$

By multiplying the left and right sides of equation (18) by $\sqrt{NKT^{(g)}}$, we get the following equations:

$$\sqrt{NKT^{(g)}}\left(\hat{\mathbf{b}}^{(g)}_{MLE,TSpVARX}-\mathbf{b}^{(g)}_{0,TSpVARX}\right)=-\left[\frac{H\left(\mathbf{b}^{(g)}_{0,TSpVARX}\right)}{NKT^{(g)}}\right]^{-1}\sqrt{NKT^{(g)}}\,\overline{S}\left(\mathbf{b}^{(g)}_{0,TSpVARX}\right), \quad (19)$$

where:

$T^{(g)}$ is the number of observations used in the g-th regime in each location and endogenous variables.

Based on the equation (16), $S\left(\hat{\mathbf{b}}^{(g)}_{MLE,TSpVARX}\right)=\mathbf{0}$. It has several consequences, namely:

$$E\left(-\left[\frac{H\left(\mathbf{b}^{(g)}_{0,TSpVARX}\right)}{NKT^{(g)}}\right]^{-1}\sqrt{NKT^{(g)}}\,\overline{S}\left(\mathbf{b}^{(g)}_{0,TSpVARX}\right)\right)=0 \quad (20)$$

and

$$Var\left(-\left[\frac{H\left(\mathbf{b}^{(g)}_{0}\right)}{NKT^{(g)}}\right]^{-1}\sqrt{NKT^{(g)}}\,\overline{S}\left(\mathbf{b}^{(g)}_{0,TSpVARX}\right)\right)=\left(-\left[\frac{H\left(\mathbf{b}^{(g)}_{0,TSpVARX}\right)}{NKT^{(g)}}\right]^{-1}\right).\left(-\frac{1}{NKT^{(g)}}E\left(H\left(\mathbf{b}^{(g)}_{0,TSpVARX}\right)\right)\right)$$
$$\times\left(-\left[\frac{H\left(\mathbf{b}^{(g)}_{0,TSpVARX}\right)}{NKT^{(g)}}\right]^{-1}\right). \quad (21)$$

When $NKT^{(g)}\rightarrow\infty$, we can get $-\frac{1}{NKT^{(g)}}H\left(\beta^{(g)}_{0,TSpVARX}\right)\rightarrow-\frac{1}{NKT^{(g)}}E\left(H\left(\beta^{(g)}_{0,TSpVARX}\right)\right)$. Because of that, equation (21) can be written as follows:

$$Var\left(-\left[\frac{H\left(\beta^{(g)}_{0,TSpVARX}\right)}{NKT^{(g)}}\right]^{-1}\sqrt{NKT^{(g)}}\,\overline{S}\left(\mathbf{b}^{(g)}_{0,TSpVARX}\right)\right)=\left(E\left(-\frac{1}{NKT^{(g)}}H\left(\mathbf{b}^{(g)}_{0,TSpVARX}\right)\right)\right)^{-1}E\left(-\frac{1}{NKT^{(g)}}H\left(\mathbf{b}^{(g)}_{0,TSpVARX}\right)\right)$$
$$\times\left(E\left(-\frac{1}{NKT^{(g)}}H\left(\mathbf{b}^{(g)}_{0,TSpVARX}\right)\right)\right)^{-1}. \quad (22)$$

Based on the Lidenberg-Levy central limit theorem (CLT), $\sqrt{NKT^{(g)}}\left(\hat{\mathbf{b}}^{(g)}_{MLE,TSpVARX}-\mathbf{b}^{(g)}_{0,TSpVARX}\right)\xrightarrow{d}N\left(0,\Sigma_{\beta}\right)$, when $NKT^{(g)}\rightarrow\infty$ (Lütkepohl, 2005).

According to the equation (20) and (22), we can get:

$$\sqrt{NKT^{(g)}}\left(\hat{\mathbf{b}}^{(g)}_{MLE,TSPVARX}-\mathbf{b}^{(g)}_{0,TSpVARX}\right)\sim N\left(\mathbf{0},\left(E\left(-\frac{1}{NKT^{(g)}}H\left(\mathbf{b}^{(g)}_{0,TSpVARX}\right)\right)\right)^{-1}E\left(-\frac{1}{NKT^{(g)}}H\left(\mathbf{b}^{(g)}_{0,TSpVARX}\right)\right)\left(E\left(-\frac{1}{NKT^{(g)}}H\left(\mathbf{b}^{(g)}_{0,TSpVARX}\right)\right)\right)^{-1}\right)$$

$$\left(\hat{\mathbf{b}}^{(g)}_{MLE,TSpVARX}-\mathbf{b}^{(g)}_{0,TSpVARX}\right)\sim N\left(\mathbf{0},\left(-E\left(H\left(\mathbf{b}^{(g)}_{0,TSpVARX}\right)\right)\right)^{-1}\right), \text{ when } NKT^{(g)}\rightarrow\infty. \quad (23)$$

Since $\beta^{(g)}_{0,TSpVARX}$ is the true value of $\beta^{(g)}_{TSpVARX}$, we can write equation (23) as the following equation:

$$\left(\hat{\beta}^{(g)}_{QMLE,TSpVARX} - \beta^{(g)}_{TSpVARX}\right) \sim N\left(\mathbf{0},\left(-E\left(H\left(\beta^{(g)}_{TSpVARX}\right)\right)\right)^{-1}\right), \text{ when } NKT^{(g)} \to \infty .$$

Let $\left(I\left(\beta_{TSpVARX}\right)\right)^{-1} = \left(-E\left(H\left(\beta^{(g)}_{TSpVARX}\right)\right)\right)^{-1}$, we get the following result:

$$\left(\hat{\beta}^{(g)}_{QMLE,TSpVARX} - \beta^{(g)}_{,TSpVARX}\right) \sim N\left(\mathbf{0},\left(I\left(\beta_{TSpVARX}\right)\right)^{-1}\right), \text{ when } NKT^{(g)} \to \infty .$$

$$\hat{\beta}^{(g)}_{QMLE,TSpVARX} \sim N\left(\beta^{(g)}_{TSpVARX},\left(I\left(\beta\right)\right)^{-1}\right)$$

Thus theorem 2 is proved.

REFERENCES

ABDULLAH, T.–WAHJUSAPUTRI, S. (2018): *Bank & Lembaga Keuangan* (2nd ed.). Mitra Wacana Media.

AMANDA, S. T.–AKHYAR, C.–ILHAM, R. N.–ADNAN (2023): The effect of inflation, exchange exchange, interest rate on stock price in the transportation sub-sector, 2018–2020 *Journal of Accounting Research, Utility Finance and Digital Assets* 1 (4): 342–352. https://doi.org/10.54443/jaruda.v1i4.54

ANSELIN, L. (1988): *Spatial econometrics: methods and models* Springer, Dordrecht. https://doi.org/10.1007/978-94-015-7799-1

ANUGRAH, D. F.–PRATAMA, R. (2018): *Core inflation determinants: decomposition of global and domestic factors associated with sectoral inflation* WP/31/2018, Bank Indonesia, Jakarta.

APRILIADARA, M.–SUHARTONO–PRASTYO, D. D. (2016): VARI-X model for currency inflow and outflow forecasting with Eid Fitr effect in Indonesia *AIP Conference Proceedings* 1746: 020041. https://doi.org/10.1063/1.4953966

BEENSTOCK, M.–FELSENSTEIN, D. (2019): Spatial vector autoregressions *The Econometric Analysis of Non-Stationary Spatial Panel Data* 129–161.

CAMILA, D.–SYAMSU, A. P.–FIRDAUS, A. D.–NORZULAIKA, S.–BINTI SULAIMAN, W. N. N. (2022): Crude palm oil corruption: a discourse on the imposition of capital punishment *Indonesian Comparative Law Review* 5 (1): 1–11. https://doi.org/10.18196/iclr.v5i1.15112

CHRISTOPOULOS, D.–MCADAM, P.–TZAVALIS, E. (2023): Threshold endogeneity in threshold VARs: an application to monetary state dependence *Federal Reserve Bank of Kansas City Working Paper* 23 (09).

CRYER, J.–CHAN, K. (2008): *Time series analysis with application in R*. Springer.

DAN HIBON, M. (1997): ARMA Models and the Box-Jenkins Methodology *Journal of Forecasting* 16 (3): 147–163. https://doi.org/10.1002/(SICI)1099-131X(199705)16:3%3C147::AID-FOR652%3E3.0.CO;2-X

DENBEL, F.–AYEN, Y.–REGASA, T. (2016): The relationship between inflation and economic growth in Ethiopia : co integration and causality analysis *International Journal of Scientific and Research Publications* 6 (1): 556–565. https://doi.org/10.33258/birci.v1i3.73

DI GIACINTO, V. (2010): On vector autoregressive modeling in space and time *Journal of Money, Credit and Banking* 42 (1): 107–141. http://onlinelibrary.wiley.com/doi/10.1111/j.1538-4616.2010.00331.x/full

ENDERS, W. (2004): *Applied econometric time series 2nd edition* John Wiley & Sons Inc., New York.

FARANDY, A. R. (2020): Analyzing factors affecting Indonesian food price inflation *Jurnal Ekonomi Dan Pembangunan* 28 (1): 65–76. https://doi.org/10.14203/jep.28.1.2020.65-76

GRANGER, C. W.–TERASVIRTA (1993): *Modelling nonlinear economic relationships (advanced texts in econometrics)* Oxford University Press, Oxford.

GUJARATI, D. N. (2003): *Basic econometrics* McGraw-Hill Companies.

HANNAN, E. J. (1970): *Multiple time series* John Wiley and Sons, Inc., New York. https://doi.org/10.1002/9780470316429

HANSEN, B. E. (1999): Testing for linearity *Journal of Economic Surveys* 13: 551–576. https://doi.org/10.1016/0167-7152(85)90011-2

HENDAYANTI, N. P. N.–NURHIDAYATI, M.–NUGRAHINI, D. S. (2017): Analisis Pengaruh Nilai Tukar Rupiah terhadap Jumlah Uang Beredar dengan Pendekatan Error Correction Model (ECM) *E-Proceedings KNS&I STIKOM Bali:* 186–190.

HUDA, E. N.–SIDIQ, S. (2023): Cooking oil price control policy In Indonesia (a review of Ibn Taimiyah's thoughts) *Reinforce: Journal of Sharia Management* 2 (1): 30–49. https://doi.org/10.21274/reinforce.v2i1.7313

IMRO'AH, N. (2023): Determination of the best weight matrix for the Generalized Space Time Autoregressive (GSTAR) model in the Covid-19 case on Java Island, Indonesia *Spatial Statistics* 54: 100734.

ISLAM, T. U.–AHMED, D. (2023). Inflation targeting: a time-frequency causal investigation *Plos one* 18 (12): e0295453.

KATMAS, E.–INDARNINGSIH, N. A. (2022): The effect of BI interest rate, exchange rate, and inflation on the Indonesian Sharia Stock Index (ISSI) *Indonesian Interdisciplinary Journal of Sharia Economics (IIJSE)* 5 (2): 769–783. http://dx.doi.org/10.31538/iijse.v5i2.2144

LIU, L. M. (1980): Note – analysis of time series with calendar effects *Management Science* 26 (1): 106–112. https://doi.org/10.1287/mnsc.26.1.106

LO, M. C.–ZIVOT, E. (2001): Threshold cointegration and nonlinear adjustment to the law of one price *Macroeconomic Dynamics* 5: 533–576. https://doi.org/10.1017/s1365100501023057

LÜTKEPOHL, H. (2005): *New introduction to multiple time series analysis* Springer, Berlin, Heidelberg. https://doi.org/10.1007/978-3-540-27752-1

MONICA, M.–SUHARSONO, A.–OTOK, B. W.–WIBISONO, A. (2021): Hybrid SARIMA-FFNN model in forecasting cash outflow and inflow *Journal of Physics: Conference Series* 2106: 012002. https://doi.org/10.1088/1742-6596/2106/1/012002

NAIROBI, N.–CAROLINE, L. (2021): Analysis of food inflation in Java and Sumatra islands *Indonesian Journal of Economics, Social, and Humanities* 3 (2): 117–129. https://doi.org/10.31258/ijesh.3.2.117-129

NUGRAHA, A. L.–JANWARI, Y.–ATHOILLAH, M. A.–MULYAWAN, S. (2023): Inflation and monetary policy: Bank Indonesia's role in suppressing the inflation rate of Islamic economic objectives *Islamic Economics and Business Review* 2 (1): 96–111.

PHILLIPS, P. C. B.–PERRON, P. (1988): Biometrika trust testing for a unit root in time series regression testing for a unit root in time series regression *Biometrika* 75 (2): 335–381. https://doi.org/10.1093/biomet/75.2.335

PRASTUTI, M.–ARIDINANTI, L.–DWININGTYAS, W. P. (2021): Spatio-temporal models with intervention effect for modelling the impact of Covid-19 on the tourism sector in Indonesia *In Journal of Physics: Conference Series* 1821 (1): 012044. IOP Publishing.

PRASTYO, D. D.–SUHARTONO, S.–PUKA, S. O. B.–LEE, M. H. (2018): Comparison between hybrid quantile regression neural network and autoregressive integrated moving average with exogenous variable for forcasting of currency inflow and outflow in Bank Indonesia *Jurnal Teknologi* 80 (6): 61–68. http://dx.doi.org/10.11113/jt.v80.11785

PRIWININGSIH, V. A.–ABIDIN, A. Z. (2022): Literature study of cooking oil scarcity and the increase of cooking oil prices in Indonesia *Proceedings of International Conference on Economics Business and Government Challenges* 1 (1): 87–92. https://doi.org/10.33005/ic-ebgc.v1i1.14

RAHMAN, I.–RATNASARI, R. T.–WARDHANA, A. K. (2022): Effect of certificate of Bank Indonesia Sharia and Indonesian Bank seven days repository rate to inflation ratio in Indonesia during Covid-19 pandemic *Economic Education and Entrepreneurship Journal* 5 (1): 157–174. https://doi.org/10.23960/e3j/v5i1.157-174

RUCHJANA, B. N.–BOROVKOVA, S. A.–LOPUHAA, H. P. (2012): Least squares estimation of Generalized Space Time AutoRegressive (GSTAR) model and its properties *AIP Conference Proceedings* 1450: 61–64. https://doi.org/10.1063/1.4724118

SIMS, C. (1977): [MT] Macro-economics and reality *Center of Economics Research, Discussion Paper No, 77–91*. Center for Economic Research , Department of Economics University of Minnesota.

SOHIBIEN, G. P. D. (2018): Analysis of the effect of fuel price policy on Jakarta inflation by using multi-input intervention model *AIP Conference Proceedings* 2014: 020125. https://doi.org/10.1063/1.5054529

SOHIBIEN, G. P. D.–LAOME, L.–CHOIRUDDIN, A.–KUSWANTO, H. (2022): Covid-19 pandemic's impact on return on asset and financing of Islamic Commercial Banks: evidence from Indonesia *Sustainability (Switzerland)* 14 (3): 1128. https://doi.org/10.3390/su14031128

SOHIBIEN, G. P. D.–SETIAWAN, S.–PRASTYO, D. D. (2024): Data forecasting performance evaluation of threshold spatial vector autoregressive with exogenous variables *International Journal of Data and Network Science* 8 (1): 523–536. https://doi.org/10.5267/j.ijdns.2023.9.004

SUHARTONO–GAZALI, M. M.–PRASTYO, D. D. (2018): VARX and GSTARX models for forecasting currency inflow and outflow with multiple calendar variations effect *Matematika* 34 (3): 57–72. https://doi.org/10.11113/matematika.v34.n3.1139

SUHARTONO–WAHYUNINGRUM, S. R.–SETIAWAN–AKBAR, M. S. (2016): GSTARX-GLS model for spatio-temporal data forecasting *Malaysian Journal of Mathematical Sciences* 10: 91–103.

SUKONO, S.–IBRAHIM, R. A.–RIAMAN, R.–HERTINI, E.–HIDAYAT, Y.–SAPUTRA, J. (2023): Forecasting the effectiveness of Covid-19 vaccination using vector autoregressive with an exogenous variable: on the cases of Covid-19 in Indonesia *Discrete Dynamics in Nature and Society*.

SUMARMININGSIH, E.–SETIAWAN, S.–SUHARSONO, A.–RUCHJANA, B. N. (2018): Spatial vector autoregressive model with calendar variation for East Java inflation and money supply *Applied Mathematics & Information Sciences* 1663: 012005. https://doi.org/10.1088/1742-6596/1663/1/012005

SUMARMININGSIH, E.–SETIAWAN, S.–SUHARSONO, A.–RUCHJANA, B. N. (2021): Comparison of spatial weights in modeling inflation and money supply using the spatial vector autoregressive model with calendarvVariations *IOP Conference Series: Materials Science and Engineering* 1115 (1): 012088. https://doi.org/10.1088/1757-899x/1115/1/012088

TONG, H. (1983): *Lecture notes in statistics, threshold models in non-linear time series analysis* (Vol. 1). Springer, New York. https://doi.org/10.1007/978-1-4684-7888-4

TSAGKANOS, A.–EVGENIDIS, A.–VARTHOLOMATOU, K. (2018): Financial and monetary stability across Euro-zone and BRICS: an exogenous threshold VAR approach *Research in International Business and Finance* 44: 386–393. https://doi.org/10.1016/j.ribaf.2017.07.108

TSAY, R. S. (1989): Testing and modeling threshold autoregressive processes *Journal of the American Statistical Association* 84 (405): 231–240. https://doi.org/10.1080/01621459.1989.10478760

UBIDE, Á. (2022): The inflation surge of 2021–22: scarcity of goods and commodities, strong labor markets and anchored inflation expectations *Intereconomics* 57 (2): 93–98. https://doi.org/10.1007/s10272-022-1036-7

YUHAN, R. J.–SOHIBIEN, G. P. D. (2018): Relationship between inflation, exchange rate and money supply in Indonesia using threshold vector autoregressive (TVAR) *AIP Conference Proceedings* 2014: 020128. https://doi.org/10.1063/1.5054532

ZHOU, X.–CHEN, Z. (2023): The impact of uncertainty shocks to consumption under different confidence regimes based on a stochastic uncertainty-in-mean TVAR model *Sustainability* 15 (4): 3032.

INTERNET SOURCES

KPPIP (2021): *Tingkatkan Konektivitas Segitiga Emas Joglosemar, Pemerintah Dorong Percepatan Pembangunan Psn Jalan Tol Solo – Yogyakarta – Kulonprogo Dan Jalan Tol Yogyakarta – Bawen.* https://kppip.go.id/berita/tingkatkan-konektivitas-segitiga-emas-joglosemar-pemerintah-dorong-percepatan-pembangunan-psn-jalan-tol-solo-yogyakarta-kulonprogo-dan-jalan-tol-yogyakarta-bawen/ (downloaded: November 2023)

WEI, W. W. S. (2006): *Time series analysis: univariate and multivariate methods* Pearson, Boston. https://civil.colorado.edu/~balajir/CVEN6833/lectures/wwts-book.pdf (downloaded: November 2023)